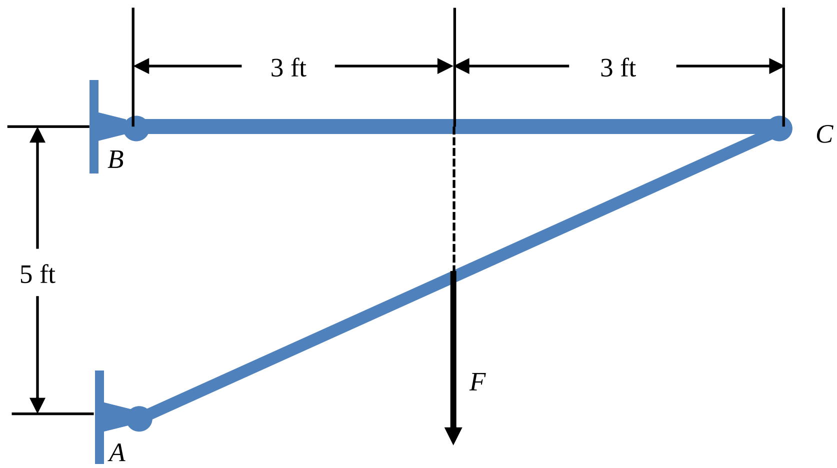
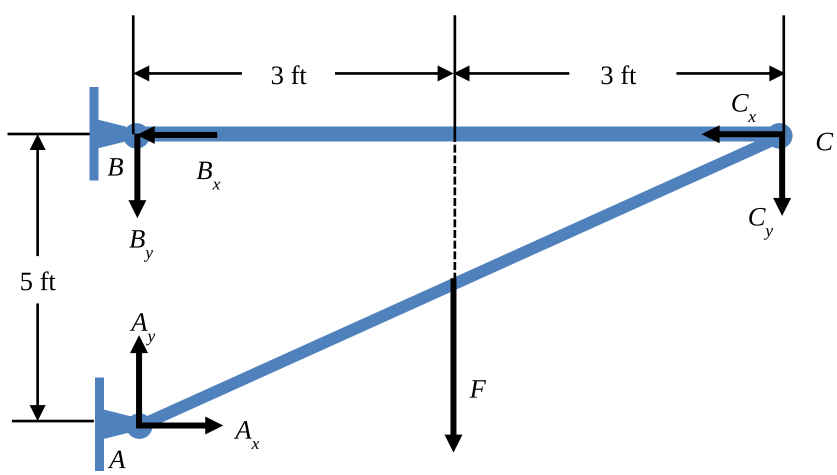
14. Pins *B* and *C* can only support horizontal loads of 130 lb. What are the probabilities of failure if $F \sim N(100, 2.5^2)$ lb.



Solution



Consider *AC*,

$$\sum M_A = 0; \quad -C_y(6) + C_x(5) - F(3) = 0,$$
$$\sum F_x = 0; \quad A_x = C_x,$$
$$\sum F_y = 0; \quad A_y = F + C_y.$$

Consider BC,

$$\sum M_B = 0; \quad -F(3) + C_y(6) = 0,$$
$$\sum F_x = 0; \quad C_x = B_x,$$
$$\sum F_y = 0; \quad B_y = C_y.$$

The external force T follows the normal distribution $N(100, 2.5^2)$ lb, thus we have

$$\mu_{C_y} = 0.5\mu_F = 50 \text{ lb},$$
$$\sigma_{C_y} = 0.5\sigma_F = 1.25,$$
$$\mu_{B_y} = 0.5\mu_F = 50 \text{ lb},$$
$$\sigma_{B_y} = 0.5\sigma_F = 1.25,$$
$$\mu_{C_x} = \frac{6}{5}\mu_F = 120 \text{ lb}$$
$$\sigma_{C_x} = \frac{6}{5}\sigma_F = 3,$$
$$\mu_{B_x} = \frac{6}{5}\mu_F = 120 \text{ lb},$$
$$\sigma_{B_x} = \frac{6}{5}\sigma_F = 3.$$

Consequently, we have the distributions of the horizontal and vertical components of forces at pins B and C. $B_x \sim N(120, 3^2)$ lb, $B_y \sim N(50, 1.25^2)$ lb, $C_x \sim N(120, 3^2)$ lb and $C_y \sim N(50, 1.25^2)$ lb. **Ans.**

The probability of pin B may fail is

$$P_B = P\left(N_{B_x} \geq 130 \text{ lb}\right) = 1 - P\left(N_{B_x} < 130 \text{ lb}\right) = 1 - \Phi\left(\frac{130-120}{3}\right) = 4.291 \times 10^{-4}. \quad \textbf{Ans.}$$

The probability of pin C may fail is also

$$P_C = P\left(N_{C_x} \geq 130 \text{ lb}\right) = 1 - P\left(N_{C_x} < 130 \text{ lb}\right) = 1 - \Phi\left(\frac{130-120}{3}\right) = 4.291 \times 10^{-4}. \quad \textbf{Ans.}$$