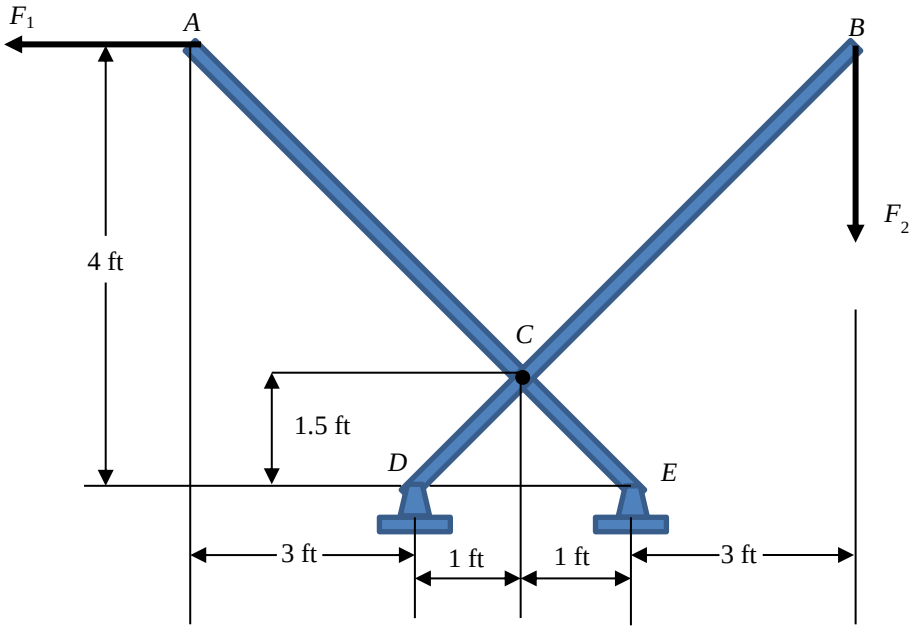
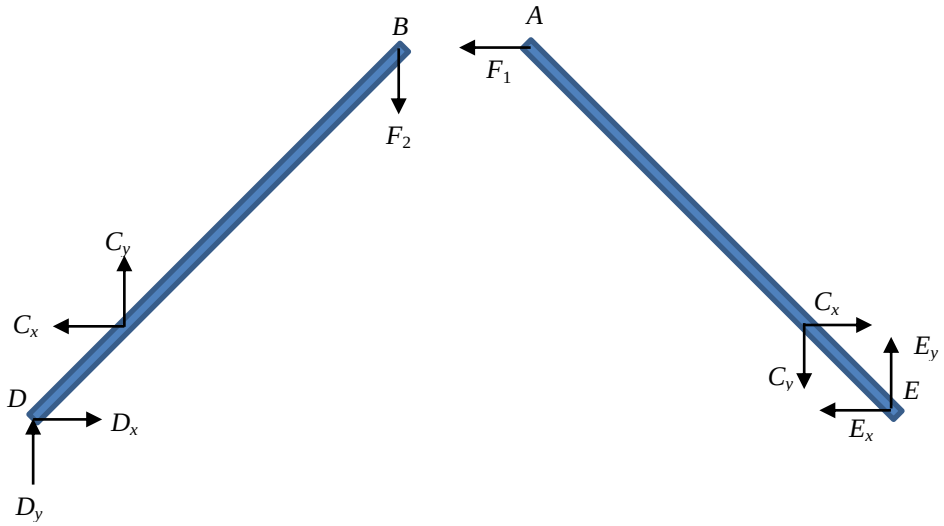
15. The two member frame is pin connected at C, D and E. Two random forces F_1 and F_2 which follow normal distributions $F_1 \sim N(150,2.5^2)$ lb and $F_2 \sim N(130,1.5^2)$ lb act horizontally at A and vertically at B respectively. Determine the distributions of horizontal and vertical reactions at each pin. If the maximum horizontal and vertical load of connector C is 450 lb and 45 lb respectively, what is the probability of failure of this system?





Solution

Member *DCB*,

$$\sum M_D = 0; \quad C_y(1) + C_x(1.5) - F_2(5) = 0.$$

Member ACE,

$$\sum M_E = 0; \quad C_y(1) - C_x(1.5) + F_1(4) = 0.$$

Solving them, we have

$$\begin{aligned}
\mu_{C_x} &= \frac{4}{3}\mu_{F_1} + \frac{5}{3}\mu_{F_2} = 416.7 \text{ lb}, \\
\sigma_{C_x} &= \sqrt{(\frac{4}{3}\mu_{F_1})^2 + (\frac{5}{3}\mu_{F_2})^2} = 3.86, \\
\mu_{C_y} &= 2.5\mu_{F_2} - 2\mu_{F_1} = 25 \text{ lb}, \\
\sigma_{C_y} &= \sqrt{(2.5\mu_{F_2})^2 + (2\mu_{F_1})^2} = 6.25 \text{ lb}.
\end{aligned}$$

Member *DCB*,

$$\begin{aligned}
\sum F_x = 0; \quad C_x = D_x, \\
\sum F_y = 0; \quad D_y + C_y = F_2.
\end{aligned}$$

Member ACE,

$$\begin{aligned}
\sum F_x = 0; \quad F_1 - C_x + E_x = 0, \\
\sum F_y = 0; \quad E_y - C_y = 0.
\end{aligned}$$

Therefore, we can obtain

$$\begin{aligned}
\mu_{D_x} &= \mu_{C_x} = 416.7 \text{ lb}, \\
\sigma_{D_x} &= \sigma_{C_x} = 4.17, \\
\mu_{D_y} &= \mu_{F_2} - \mu_{C_y} = 105 \text{ lb}, \\
\sigma_{D_y} &= \sqrt{\mu^2_{F_2} + \mu^2_{C_y}} = 6.43, \\
\mu_{E_x} &= \mu_{C_x} - \mu_{F_1} = 266.7 \text{ lb}, \\
\sigma_{E_x} &= \sqrt{\mu^2_{C_x} + \mu^2_{F_1}} = 4.86, \\
\mu_{E_y} &= \mu_{C_y} = 25 \text{ lb}, \\
\sigma_{E_y} &= \sigma_{C_y} = 6.25.
\end{aligned}$$

Finally, we obtain the distributions of all the components on *C*, *D* and *E*: $C_x \sim N(416.7, 4.17^2)$ lb, $C_y \sim N(25, 6.25^2)$ lb, $D_x \sim N(416.7, 4.17^2)$ lb, $D_y \sim N(105, 6.43^2)$ lb, $E_x \sim N(266.7, 4.86^2)$ lb and $E_y \sim N(25, 6.25^2)$ lb. **Ans.**

The maximum horizontal and vertical load of connector *C* is 450 lb and 45 lb respectively, therefore the probability of connector *C* to fail in the horizontal direction is

$$P_h\left(C_x \geq 450 \text{ lb}\right) = 1 - P_h\left(C_x < 450 \text{ lb}\right) = 1 - \Phi\left(\frac{450 - 416.7}{4.17}\right) = 0, \quad \textbf{Ans.}$$

And the probability of connector *C* to fail in the vertical direction is

$$P_V\left(C_y \geq 45 \text{ lb}\right) = 1 - P_V\left(C_y < 45 \text{ lb}\right) = 1 - \Phi\left(\frac{45 - 25}{6.25}\right) = 6.8714 \times 10^{-4}. \quad \textbf{Ans.}$$

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