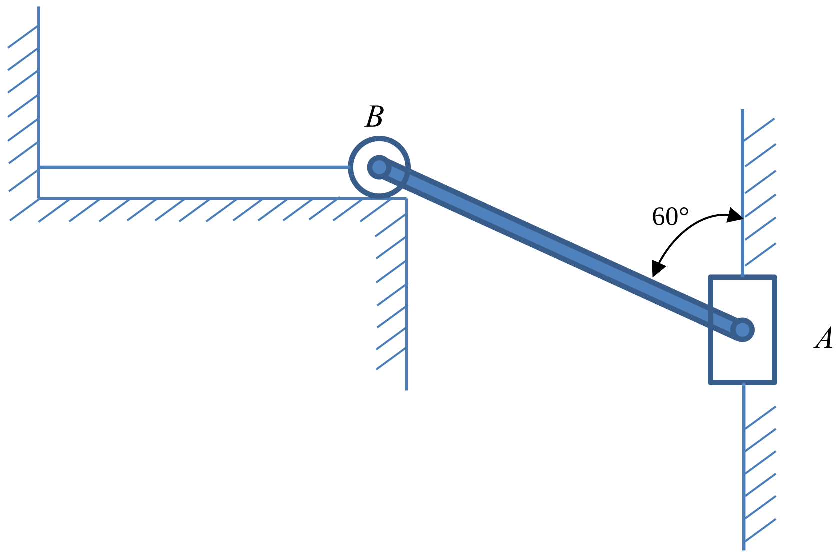
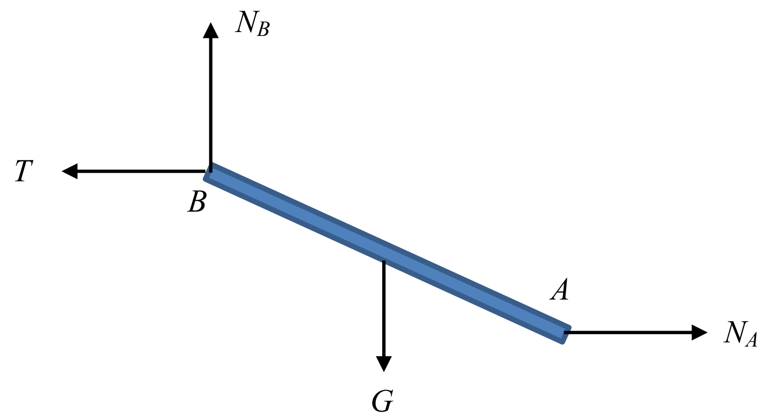
16. The uniform rod AB with a length 6 ft has a normally distributed weight $G \sim N(18,\ 0.2^2)$ lb. (1) Determine the distribution of the force in the cable when the rod is in the position shown. (2) The maximum tension of the rod follows $S \sim N(17, 0.4^2)$ lb, what is the probability the system might fail if G and S are independent?



Solution



(1)

$$\sum F_y = 0; \quad N_B = G$$

Since the weight follows the normal distribution $G \sim N(18,\ 0.2^2)$ lb, we can obtain

$$\mu_B = \mu_G = 18\,\text{lb}$$

$$\sigma_B = \sigma_G = 0.2$$

Also, we have

$$\sum M_A = 0; \quad N_B(6)(\sin 60^\circ) - G(3)(\sin 60^\circ) - T(6)(\cos 60^\circ) = 0$$

Thus, we can get the force in the cable

$$\mu_T = \sqrt{3}\mu_B - \frac{\sqrt{3}}{2}\mu_G = 15.6 \text{ lb.}$$

$$\sigma_T = \sqrt{3\sigma^2{}_B + \frac{3}{4}\sigma^2{}_G} = 0.39$$

Therefore, the force in the cable follows the distribution $T \sim N(15.6,\ 0.39^2)$ lb. **Ans**.

(2)

If the maximum load of the cable follows distribution $S \sim N(17, 0.4^2)$ lb, then we can construct

$$Y = T - S$$

$$\mu_Y = \mu_T - \mu_S = -1.41 \text{ lb}$$

$$\sigma_Y = \sqrt{\sigma_T^2 + \sigma_S^2} = 0.557$$

Finally the probability the system may fall is

$$P(Y \geq 0) = 1 - P(Y < 0) = 1 - \Phi(\frac{1.41}{0.557}) = 0.0056$$ **Ans.**