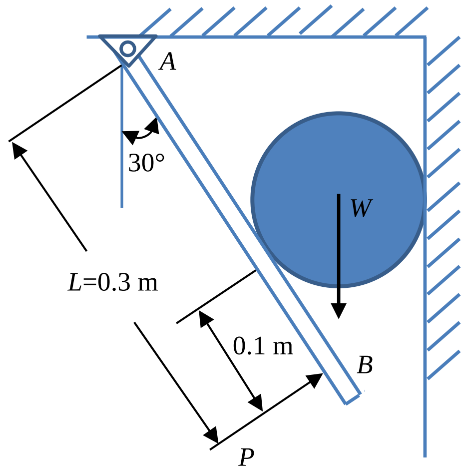
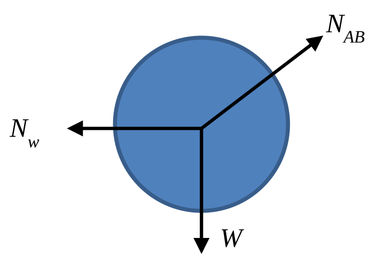
21. Plate *AB* is used to hold the ball. If the weight of the ball *W* and the applied force *P* exerted on the plate are independently and normally distributed with $W \sim N(4,\ 0.05^2)$ kg and $P \sim N(55,\ 0.5^2)$ N, respectively, determine the probability that the ball will fall. The wall is assuming to be smooth.



Solution



According to the force equilibrium on the ball, we have

$$W = N_{AB} \sin 30^\circ$$

Thus, we could obtain

$$\mu_{N_{AB}} = \frac{\mu_W (9.81)}{\sin 30^\circ} = 78.48 \text{ N}$$

$$\sigma_{N_{AB}} = \frac{\sigma_W (9.81)}{\sin 30^\circ} = 0.981$$

$$N_{AB} \sim N(78.48, 0.981^2) \text{ N}$$

The probability that the ball might fall is

$$\sum M_A \le 0; \quad P(0.3) \le N_{AB}(0.2)$$

Then, we may construct Y as below and the probability constraint becomes $Y \le 0$

$$Y = 0.3P - 0.2N_{AB}$$

Therefore, we have

$$\mu_Y = 0.3\mu_P - 0.2\mu_{N_{AB}} = 0.804 \text{ N}$$

$$\sigma_Y = \sqrt{(0.3\sigma_P)^2 + (0.2\sigma_{N_{AB}})^2} = 0.247$$

Consequently, the probability the ball might fall is

$$P(Y \le 0) = \Phi(\frac{-0.804}{0.247}) = 0.06\%$$ **Ans**.