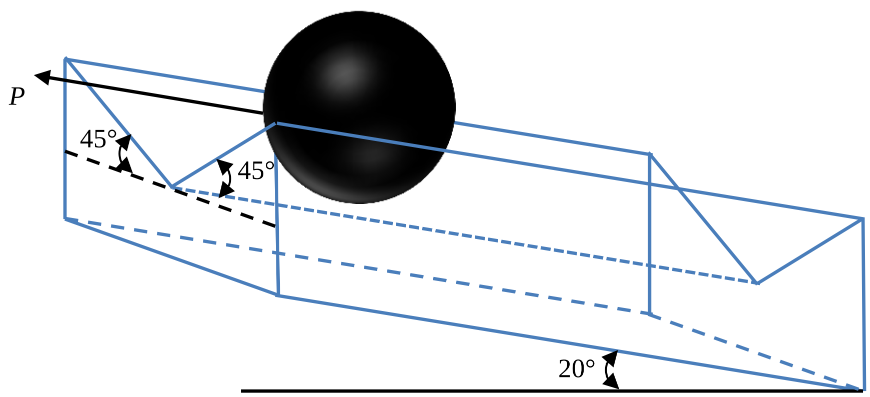
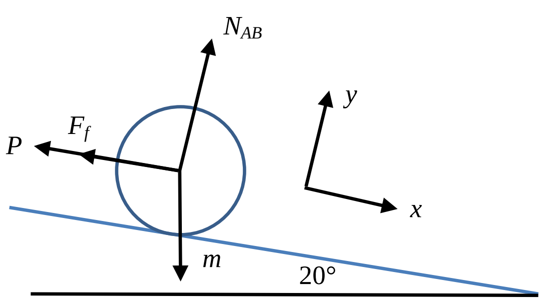
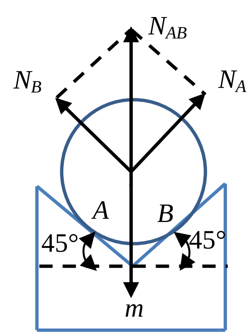
24. A ball with a weight of $m \sim N(10,\ 0.02^2)$ kg is placed between the 45° grooves *A* and *B* of the 20° incline. Determine the probability that the external force $P \sim N(7.8,\ 0.05^2)$ N which is independent from *m* may hold the ball up from slipping if the coefficient of friction between the ball and groove is $\mu_s = 0.2$.







Solution

$$\sum F_y = 0; \quad mg\cos 20^\circ = N_{AB}$$

Also, we have

$$N_A = N_B = \cos 45^\circ N_{AB}$$

$$F_f = 2\mu_s N_A = 2\mu_s N_B$$

Then, we can obtain

$$\mu_{F_f} = 2g\mu_s \cos 45^\circ \cos 20^\circ \mu_m = 26.07\text{ N}$$

$$\sigma_{F_f} = 2g\mu_s \cos 45^\circ \cos 20^\circ \sigma_m = 0.052$$

Thus, we have

$$F_f \sim N(26.07, 0.052^2) \text{ N}$$

Also, along *x* axis we have

$$\sum F_x = 0; \quad P + F_f \geq mg \sin 20^\circ$$

The probability that external force *P* can hold the ball is that *P* should satisfy the above inequality, therefore we construct

$$Y = P + F_f - mg \sin 20^\circ$$

$$\mu_Y = \mu_P + \mu_{F_f} - \mu_m g \sin 20^\circ = 0.3241 \text{ N}$$

$$\sigma_Y = \sqrt{\sigma_P^2 + \sigma_{F_f}^2 + (\sigma_m g \sin 20^\circ)^2} = 0.0985$$

$$Y \sim N(0.3241\, 0.0985^2) \text{ N}$$

Finally, the probability that the external force *P* can hold the ball is

$$P(Y \geq 0) = 1 - P(Y < 0) = \Phi(\frac{-0.3241}{0.0985}) = 99.94\%$$ **Ans**.