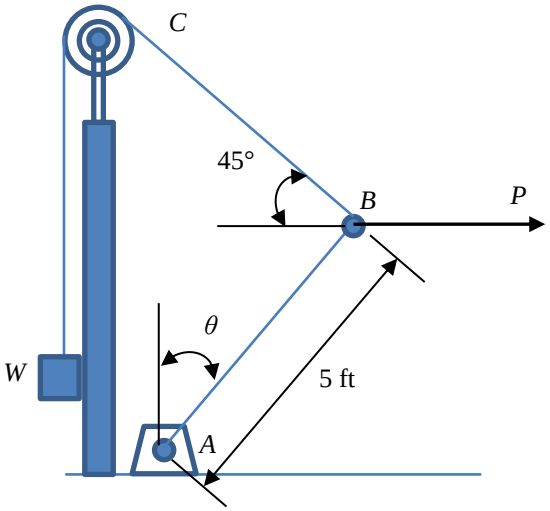
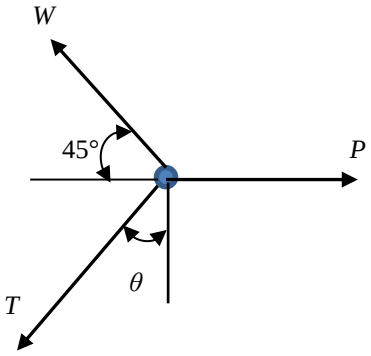
28. Two independently normally distributed weight $m \sim N(10, 0.5^2)$ kg and force $P \sim N(100, 1.5^2)$ N are shown in the system. If the cable *AB* is in the position shown below, determine the probability that the system may fail if the allowable tension of *AB* is 85 N.



Solution



$$\sum F_x = 0; \quad P - mg\cos 45^\circ - T\sin\theta = 0;$$

$$\sum F_y = 0; \quad mg\sin 45^\circ - T\cos\theta = 0;$$

Solve the above equations, we have

$$\theta = \arctan\left(\frac{P - mg\cos 45^\circ}{mg\sin 45^\circ}\right) = 23.83^\circ$$

$$T = mg\sin 45^\circ / \cos\theta$$

$$\mu_T = \mu_m g\sin 45^\circ / \cos\theta = 75.83 \text{ N}$$

$$\sigma_T = \sigma_m g\sin 45^\circ / \cos\theta = 3.79$$

Therefore, we have

$$T \sim N(75.83, 3.79^2) \quad \text{N}$$

The maximum tension of cable *AB* is 80 N, thus the probability that the system might fail is $P(Y > 0)$, where $Y = T - T_{\text{Max}}$

Finally, the probability is

$$P(Y > 0) = 1 - P(Y \le 0) = 1 - \Phi(\frac{-\mu_Y}{\sigma_Y}) = 0.78\%$$ **Ans**.