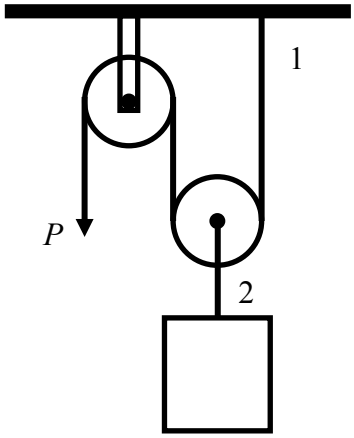
31. A frictionless pulley system, which lifts a box, is shown in the figure. The weight of the box follows a normal distribution $W \sim N(1500, 160^2)$ kN . The resistances of the two cables follow distributions $S_1 \sim N(1200, 100^2)$ kN and $S_2 \sim N(2500, 250^2)$ kN . Determine the probabilities of failure of the cables. W, S_1, and S_2 are independently distributed.



Solution

From the figure, we have

$$T_1 = 0.5W$$

$$T_2 = W$$

Then we construct the functions

$$Y_1 = S_1 - T_1 = S_1 - 0.5W$$

$$Y_2 = S_2 - T_2 = S_2 - W$$

From which, we have

$$\mu_{Y_1} = \mu_{S_1} - 0.5\mu_W = 450 \text{ kN}$$

$$\sigma_{Y_1} = \sqrt{\sigma_{S_1}^2 + \left(0.5\sigma_W\right)^2} = 128.06 \text{ kN}$$

$$\mu_{Y_2} = \mu_{S_2} - \mu_W = 1000 \text{ kN}$$

$$\sigma_{Y_2} = \sqrt{\sigma_{S_2}^2 + \left(\sigma_W\right)^2} = 296.82 \text{ kN}$$

Thus, the probabilities of failure of cable 1 and cable 2 are

$$p_{f1} = \Pr(Y_1 < 0) = 2.2078 \times 10^{-4} \quad \textbf{Ans.}$$

$$p_{f2} = \Pr(Y_2 < 0) = 3.7709 \times 10^{-4} \quad \textbf{Ans.}$$