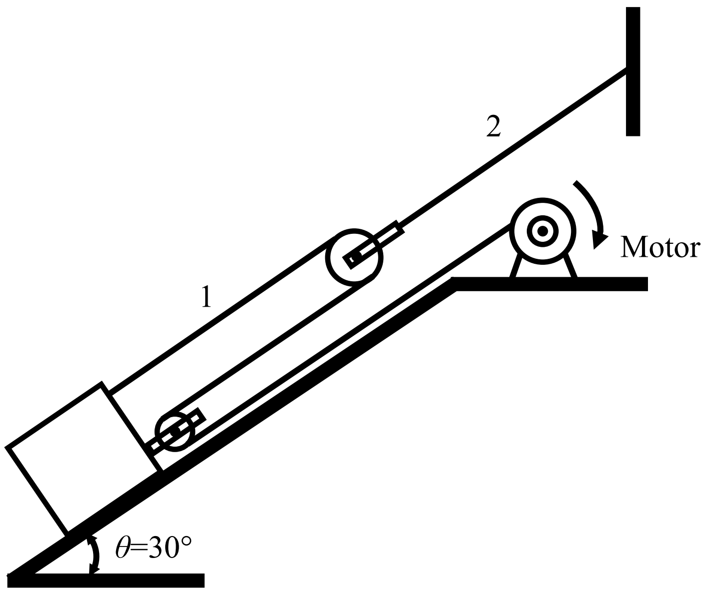
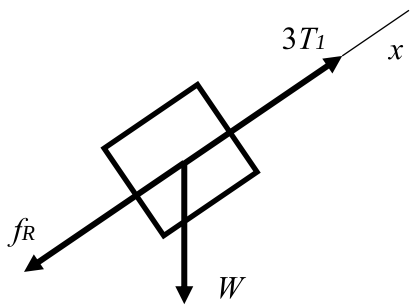
32. A frictionless pulley system, which lifts a box, is shown in the figure. The weight of the box follows a normal distribution $W \sim N(1500, 160^2)$ kN. The coefficient of friction between the box and the surface τ is 0.2. The resistances of the two cables follow distributions $S_1 \sim N(600, 65^2)$ kN and $S_2 \sim N(1220, 140^2)$ kN. Determine the probabilities of failure of the cables. W, S_1, and S_2 are independently distributed.



Solution



$$\sum F_x = 0;\ 3T_1 - W(\sin\theta + \tau\cos\theta) = 0$$

Then we have

$$T_1 = \frac{W(\sin\theta + \tau\cos\theta)}{3}$$

$$T_2 = 2T_1 = \frac{2W(\sin\theta + \tau\cos\theta)}{3}$$

Then we construct the functions

$$Y_1 = S_1 - T_1 = S_1 - \frac{W(\sin\theta + \tau\cos\theta)}{3}$$

$$Y_2 = S_2 - T_2 = S_2 - \frac{2W(\sin\theta + \tau\cos\theta)}{3}$$

From which, we have

$$\mu_{Y_1} = \mu_{S_1} - \frac{\mu_W(\sin\theta + \tau\cos\theta)}{3} = 263.4 \text{ kN}$$

$$\sigma_{Y_1} = \sqrt{\sigma_{S_1}^2 + \left(\frac{\sigma_W(\sin\theta + \tau\cos\theta)}{3}\right)^2} = 74.26 \text{ kN}$$

$$\mu_{Y_2} = \mu_{S_2} - \frac{2\mu_W(\sin\theta + \tau\cos\theta)}{3} = 546.8 \text{ kN}$$

$$\sigma_{Y_2} = \sqrt{\sigma_{S_2}^2 + \left(\frac{2\sigma_W(\sin\theta + \tau\cos\theta)}{3}\right)^2} = 157.3 \text{ kN}$$

Thus, the probabilities of failure of cable 1 and cable 2 are

$$p_{f1} = \Pr(Y_1 < 0) = 1.9475 \times 10^{-4} \quad \textbf{Ans.}$$

$$p_{f2} = \Pr(Y_2 < 0) = 2.5523 \times 10^{-4} \quad \textbf{Ans.}$$