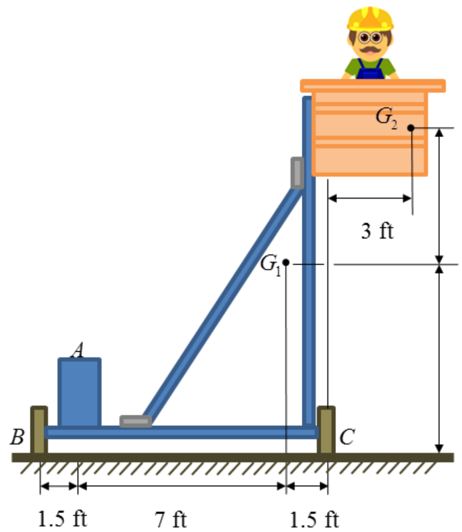
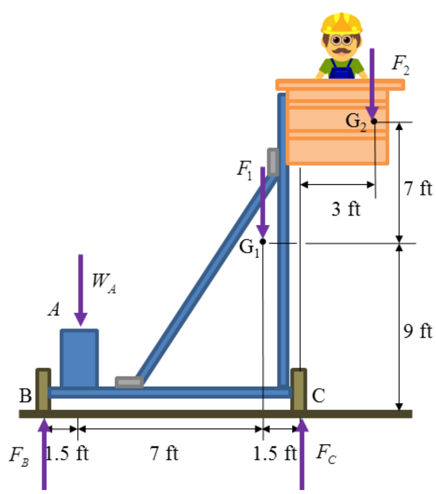
5. The weight of the platform assembly follows a normal distribution $F_1 \sim N(300, 5^2)$ lb . The center of gravity of the platform is at G_1. The counterweight W_A at A is used to prevent the platform from tippling over, and its distribution is $W_A \sim N(80, 2^2)$ lb. If the load at point G_2 is 390 lb, what is the probability that the platform will tip over? F_1 and W_A are independently distributed.



Solution



When tipping occurs, $F_B = 0$

$$\sum M_C = 0$$
$$-F_2(3) + F_1(1.5) + W_A(8.5) = 0$$

From above equation, we have

$$F_2 = \frac{F_1(1.5) + W_A(8.5)}{3}$$

Given $F_1 \sim N(300, 5^2)$ lb, $W_A \sim N(80, 2^2)$ lb and they are distributed independently, μ_{F_2} and σ_{F_2} are calculated by

$$\mu_{F_2} = \frac{\mu_{F_1}(1.5) + \mu_{W_A}(8.5)}{3} = 376.67 \text{ lb}$$

$$\sigma_{F_2} = \frac{\sqrt{\sigma_{F_1}{}^2(1.5)^2 + \sigma_{W_A}{}^2(8.5)^2}}{3} = 6.19 \text{ lb}$$

Thus, we get $F_2 \sim N(376.67, 6.19^2)$ lb

If the platform will tip over, F_2 will be greater than the maximum load 390 lb at point G_2, Suppose

$$Y = 390 - F_2$$

We can calculate μ_Y and σ_Y by

$$\mu_Y = 390 - \mu_{F_2} = 13.33 \text{ lb}$$

$$\sigma_Y = \sigma_{F_2} = 6.19 \text{ lb}$$

Thus the probability that the platform will tip over is

$$P(Y < 0) = \Phi\left(\frac{0 - \mu_Y}{\sigma_Y}\right) == 1 - \Phi\left(\frac{3.33}{6.19}\right) = 0.0156$$ **Ans.**