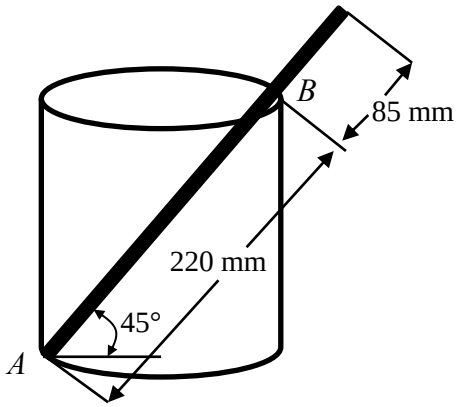
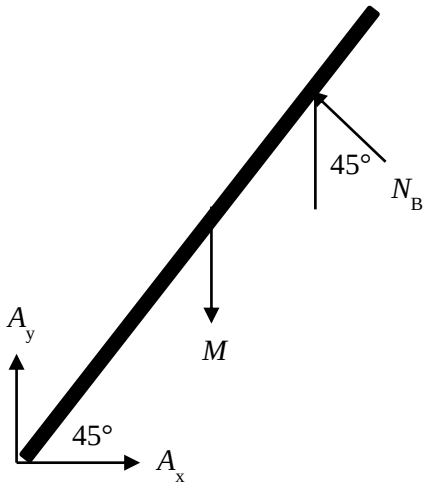
7. Rod has a mass which follows a normal distribution $M \sim N(40, 1.5^2)$ g. Determine the distributions of reactions on the rod.



Solution



$$\sum M_A = 0; \quad N_B(0.22) - M(9.81)\big((0.22+0.085)/2\big)\cos 45^\circ = 0;$$
$$\sum F_x = 0; \quad A_x - N_B \sin 45^\circ = 0;$$
$$\sum F_y = 0; \quad A_y - M(9.81) + N_B \cos 45^\circ = 0.$$

Since the rod mass follows the normal distribution $M \sim N(0.04, 0.0015^2)$ kg, we have

$$\mu_{N_B} = \frac{\mu_M (9.81)(0.305/2)\cos 45^\circ}{0.22} = 0.19 \text{ N}$$
$$\sigma_{N_B} = \frac{\sigma_M (9.81)(0.305/2)\cos 45^\circ}{0.22} = 7.2\times 10^{-3}$$

$$\mu_{A_x} = \mu_{N_B} \sin 45^\circ = 0.19 \sin 45^\circ = 0.14 \text{ N}$$
$$\sigma_{A_x} = \sigma_{N_B} \sin 45^\circ = 7.2\times 10^{-3} \sin 45^\circ = 5.1\times 10^{-3}$$

After known the distributions of N_B and A_x, we have

$$\mu_{A_y} = \mu_M (9.81) - \mu_{N_B} \cos 45^\circ = 0.26 \text{ N}$$
$$\sigma_{A_y} = \sqrt{\left(\sigma_{N_B} \cos 45^\circ\right)^2 + \left(\sigma_M (9.81)\right)^2} = 1.6\times 10^{-2}$$

Thus, the support distributions respectively as, $N_B \sim N(0.19, (7.2\times 10^{-3})^2)$ N, $A_x \sim N(0.14, (5.1\times 10^{-3})^2)$ N and $A_y \sim N(0.26, (1.6\times 10^{-2})^2)$ N. **Ans.**