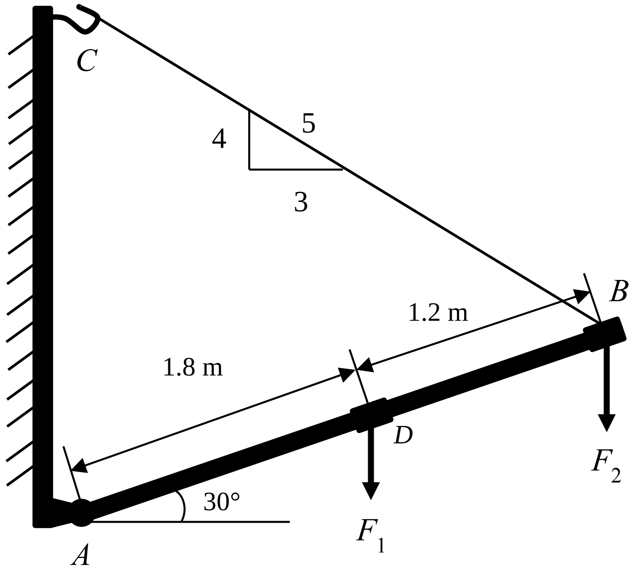
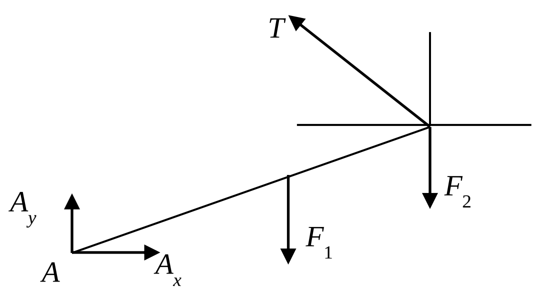
9. Rod *AB* supports two vertical loads, F_1 and F_2, which independently follow normal distributions as $F_1 \sim N(1500, 50^2)$ lb and $F_2 \sim N(600, 25^2)$ lb. (1) What is the distribution of the load on cable *CB*? (2) If cable *CB* can sustain a maximum load of 1400N before it fails, what is the probability that the cable may fail?



Solution



(1)

$$\sum M_A = 0; \quad -F_1(1.8\cos 30^\circ) - F_2(3\cos 30^\circ) + \frac{3}{5}T_{\max}(3\sin 30^\circ) + \frac{4}{5}T_{\max}(3\cos 30^\circ) = 0$$

From above equation, we have

$$T_{\max} = \frac{F_1(1.8\cos 30^\circ) + F_2(3\cos 30^\circ)}{\frac{3}{5}(3\sin 30^\circ) + \frac{4}{5}(3\cos 30^\circ)}$$

With $F_1 \sim N(1500, 50^2)$ lb and $F_2 \sim N(600, 25^2)$ lb,

$$\mu_{T_{\max}} = \frac{\mu_{F_1}(1.8\cos 30^\circ) + \mu_{F_2}(3\cos 30^\circ)}{\frac{3}{5}(3\sin 30^\circ) + \frac{4}{5}(3\cos 30^\circ)} = 1308.4 \text{ lb}$$

$$\sigma_{T_{\max}} = \frac{\sqrt{\left(\sigma_{F_1}(1.8\cos 30^\circ)\right)^2 + \left(\sigma_{F_2}(3\cos 30^\circ)\right)^2}}{\frac{3}{5}(3\sin 30^\circ) + \frac{4}{5}(3\cos 30^\circ)} = 34.06$$

Thus, the distribution of load of cable *CB* is: $T_{\max} \sim N(1308.4, 34.06^2)$ lb. **Ans.**

(2) The probability of failure is

$$P(Y \geq 1400) = 1 - P(Y < 1400) = 1 - \Phi\left(\frac{1400 - 1308.4}{34.06}\right) = 0.0036$$ **Ans.**