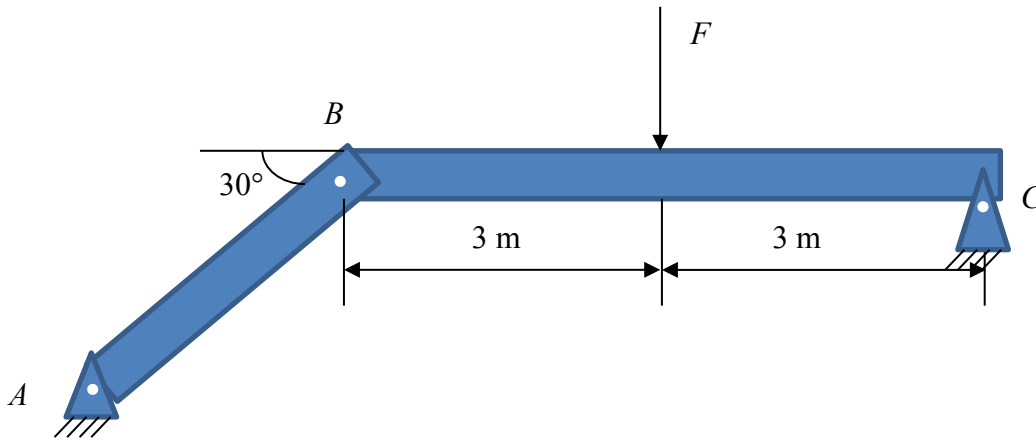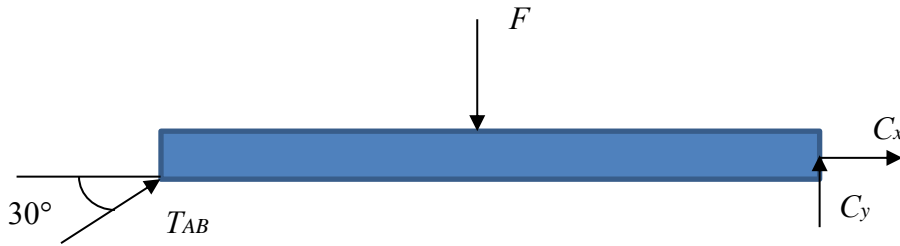11. Force F follows a normal distribution $F \sim N(800, 9^2)$ N. What is the probability that member AB will fail if the maximum normal force in AB follows $T \sim N(850, 15^2)$ N ? F and T are independently distributed.

Solution

$$\sum M_C = 0; \quad -F(3) + T_{AB} \sin 30^\circ (6) = 0$$

$$T_{AB} = F$$

Since $F \sim N(1000, 10^2)$ N, the distribution of T_{AB} is

$$T_{AB} \sim N(1000, 10^2) \text{ N}$$

We construct the function $Y = T - T_{AB}$

$$\mu_Y = \mu_T - \mu_{T_{AB}} = 50 \text{ N}$$

$$\sigma_Y = \sqrt{\sigma_{T_{AB}}^2 + \sigma_T^2} = 17.5 \text{ N}$$

If $Y = T - T_{AB} < 0$, AB fails

$$p_f = \Pr(Y < 0) = \Pr(-\mu_Y / \sigma_Y) = 0.0021 \qquad \textbf{Ans.}$$