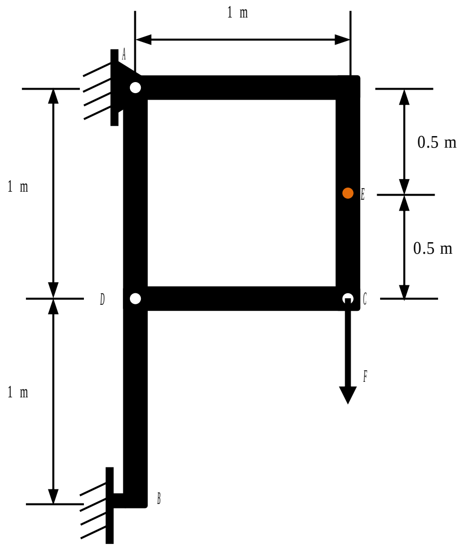
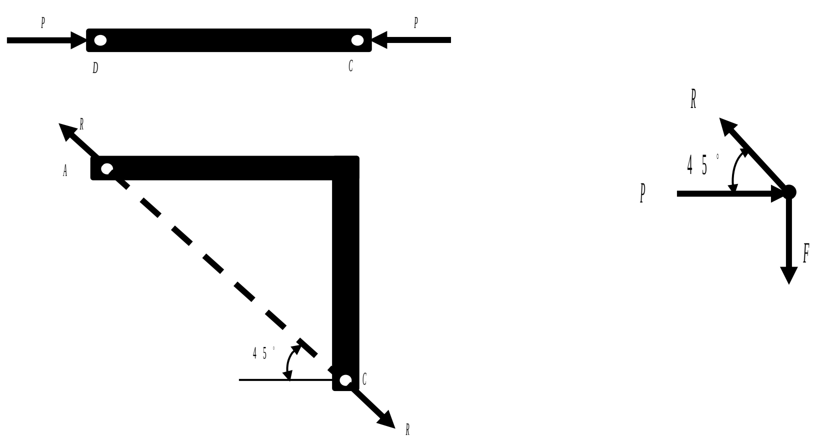
5. A force $F \sim N(800, 30^2)$ N acts at point *C*. Determine the probability of failure of the pin at *A* if the allowable is $S \sim N(1200, 25^2)$ N. *F* and *S* are independent.



Solution

The free-body diagram of each force member is shown below.



For point C,

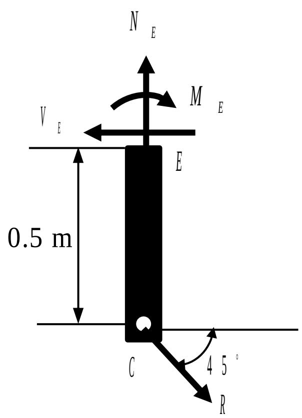
$$\Sigma F_y = 0;\ R\sin 45^\circ - F = 0$$

$$\mu_R = \frac{\mu_F}{\sin 45^\circ} = \frac{800}{\sin 45^\circ} = 1131.5 \text{ N}$$

$$\sigma_R = \frac{\sigma_F}{\sin 45^\circ} = \frac{30}{\sin 45^\circ} = 42.43$$

Thus, the distribution of force R is: $R \sim N(1131.5, 42.43^2)$ N.

The free-body diagram of segment CE is shown below.



$$\Sigma F_x = 0;\ \ -V_E + R\cos 45^\circ = 0$$
$$\Sigma F_y = 0;\ N_E - R\sin 45^\circ = 0$$
$$\Sigma M_E = 0;\ M_E - R\cos 45^\circ (0.5) = 0$$

With $R \sim N(1131.5, 42.43^2)$ N, we obtain the distributions of normal force N_E, shear force V_E, and bending moment M_E:

$$N_E \sim N(800, 30^2) \text{ N}$$ **Ans.**

$$V_E \sim N(800, 30^2) \text{ N}$$ **Ans.**

$$M_E \sim N(400, 15^2) \text{ N.m}$$ **Ans.**

However, these results indicate a poor design. Member AC should be straight (from A to C) so that bending within the member is eliminated. If AC is straight then the internal force would only create tension in the member.

Since the allowable load at pin A is $S \sim N(1200, 25^2)$, we can construct

$$Y = R - S.$$

Then we can obtain the distribution of $Y \sim N(-68.5, 49.2^2)$ N.

Therefore, the probability of failure can be calculated as

$$P(Y \geq 0) = 1 - P(Y < 0) = 1 - \Phi(\frac{68.5}{49.2}) = 0.0819.$$ **Ans.**