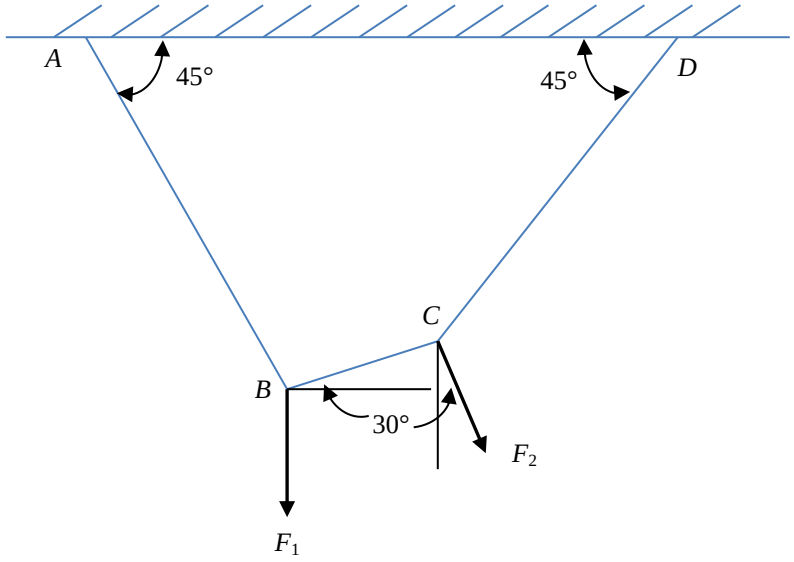
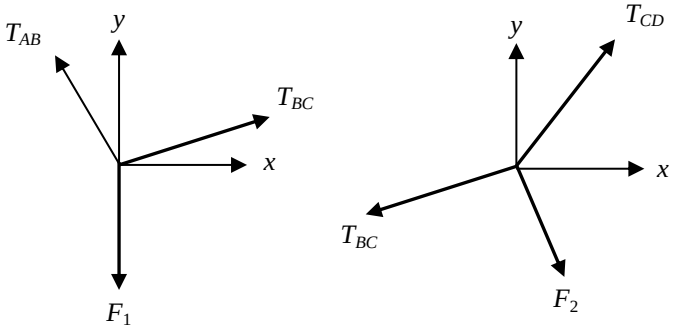
7. The cable supports two independently and normally distributed forces $F_1 \sim N(15, 2^2)$ Nand $F_2 \sim N(10, 1^2)$ N shown below. If the maximum tension of the cable is 20 N, determine the reliability of the system.



Solution



Connector *B*:

$$\sum F_x = 0; \quad T_{BC}\cos 30^\circ = T_{AB}\cos 45^\circ$$

$$\sum F_y = 0; \quad T_{BC}\sin 30^\circ + T_{AB}\sin 45^\circ = F_1$$

Solve the above equations, we have

$$T_{AB} = 0.897F_1$$

$$T_{BC} = 0.732F_1$$

Connector *C*:

$$\sum F_x = 0; \quad T_{BC}\cos 30^\circ = T_{CD}\cos 45^\circ + F_2\cos 45^\circ$$

Solve it, we have

$$T_{CD} = 0.897F_1 - F_2$$

Therefore, we can obtain the distributions of all forces

$$T_{AB} \sim N(13.45, 1.79^2)\ \text{N}, T_{BC} \sim N(10.98, 1.464^2)\ \text{N and}\ T_{CD} \sim N(3.46, 2.05^2)\ \text{N}$$

The reliability can be determined by if the largest force is smaller than the allowable tension, thus we have:

$$R = P(Y \le 0)\text{, where } Y = F_{AB} - 20\text{.}$$

Finally, we have $R = 99.99\%$ **ANS**.