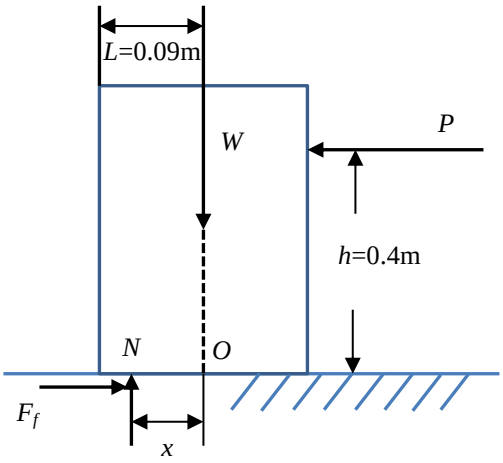
10. A normally distributed external force $P \sim N(10, 0.05^2)$ lb is applied to push a box which has a weight of 45 lb. Determine the probability that the box may tip over if the coefficient of friction between the box and floor is $\mu_s = 0.2$.



Solution

$$\sum M_O = 0; \quad Ph = xN = xW$$

Then, we have

$$\mu_x = \mu_P h / W = 0.089 \text{ m}$$

$$\sigma_x = \sigma_P h / W = 0.0004$$

$$x \sim N(0.089, 0.0004^2) \text{ m}$$

Also, we have

$$F_f = \mu_s N = 9 < P$$

The probability that the box may tip over is

$$x > L$$

Thus, we can construct and the box might tip over when Y>0

$$Y = x - L$$

$$\mu_Y = \mu_x - L = -0.0011 \text{ m}$$

$$\sigma_Y = \sigma_x = 0.0004$$

Finally, we can obtain

$$P(Y \geq 0) = 1 - P(Y < 0) = \Phi(\frac{0.0011}{0.0004}) = 0.62\% \qquad \textbf{Ans}.$$