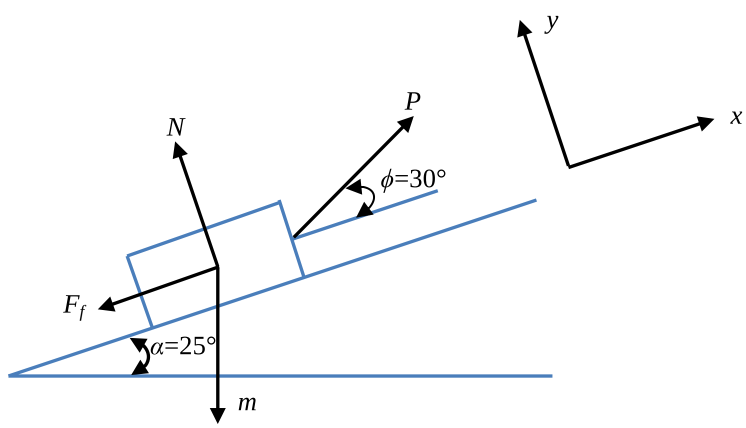
11. The block which has a normally distributed weight $m \sim N(10,\ 0.05^2)$ kg is being pulled up the inclined plane of slope α using another independent normally distributed external force $P \sim N(100,\ 1.5^2)$ N. Determine the probability that the external force P can pull the block up if the coefficient of friction is $\mu_s = 0.45$.



Solution

$$\sum F_y = 0; \quad N = mg\cos\alpha$$

And, we have

$$F_f = \mu_s N$$

Then, we can obtain

$$\mu_{F_f} = \mu_s \mu_m g\cos\alpha = (0.45)(10)(9.81)\cos 25^\circ = 40.01 \text{ N}$$

$$\sigma_{F_f} = \mu_s \sigma_m g\cos\alpha = (0.45)(0.05)(9.81)\cos 25^\circ = 0.2$$

$$F_f \sim N(40.01, 0.2^2) \text{ N}$$

If P is sufficiently large to pull the block up, then we have

$$\sum F_x \geq 0; \quad P\cos\phi \geq F_f + mg\sin\alpha$$

Therefore, we can define

$$Y = P\cos\phi - F_f - mg\sin\alpha$$

Thus, we can obtain

$$\mu Y = \mu_P \cos\phi - \mu_{F_f} - \mu_m g \sin\alpha = 5.135 \text{ N}$$

$$\sigma Y = \sqrt{(\sigma_P \cos\phi)^2 + \sigma_{F_f}^2 + (\sigma_m g \sin\alpha)^2} = 1.331$$

Finally, we can calculate the probability that the force *P* can pull the block up is

$$P(Y \geq 0) = 1 - P(Y < 0) = \Phi(\frac{-5.135}{1.331}) = 99.99\%$$ **Ans**.