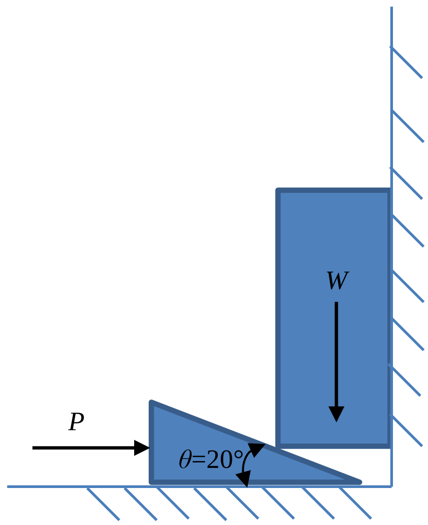
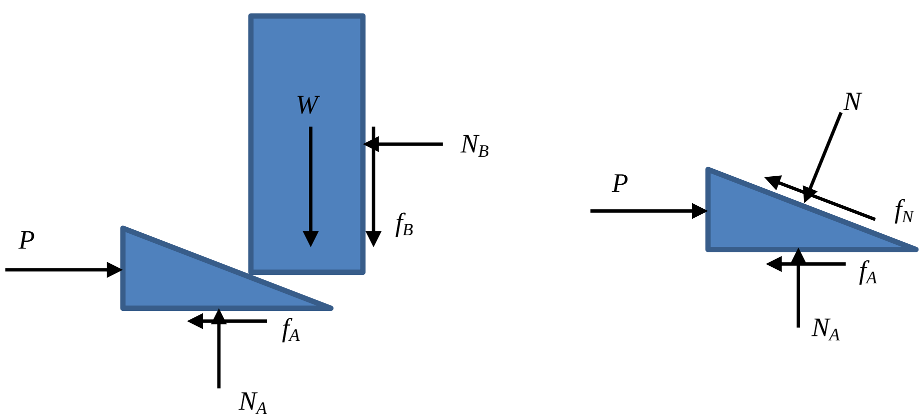
13. The coefficient of static friction between all the surfaces is $\mu_s = 0.3$, A random force $P \sim N(15,\ 1^2)$ N is applied to the wedge, and the weight of the block is $W \sim N(10,\ 0.5^2)$ N due to the manufacturing uncertainty. If P and W are independent, what is the probability that the block will be lifted.



Solution



System:

$$\sum F_x = 0; \quad P - N_B - \mu_s N_A = 0; \tag{1}$$

$$\sum F_y = 0; \quad N_A - \mu_s N_B - W = 0; \tag{2}$$

Wedge:

$$\sum F_x = 0; \quad P - \mu_s N_A - \mu_s N\cos\theta - N\sin\theta = 0; \tag{3}$$

$$\sum F_y = 0; \quad N_A - N\cos\theta + \mu_s N\sin\theta = 0; \tag{4}$$

From Eqns. (3) and (4), we obtain

$$N = \frac{P - \mu_s N_A}{\mu_s \cos\theta + \sin\theta}$$

$$N_A = \frac{P(1-\mu_s \tan\theta)}{2\mu_s + (1-\mu_s^2)\tan\theta} \tag{5}$$

From Eqns. (1) and (2)

$$N_B = P - \mu_s N_A$$

$$N_A = \frac{P\mu_s + W}{1+\mu_s^2} \tag{6}$$

Combining Eqns. (5) and (6), we have

$$P = W\left(\frac{(1-\mu_s^2)\tan\theta + 2\mu_s}{1-2\mu_s\tan\theta - \mu_s^2}\right)$$

Therefore, we have

$$\mu_P = \mu_W\left(\frac{(1-\mu_s^2)\tan\theta + 2\mu_s}{1-2\mu_s\tan\theta - \mu_s^2}\right) = 13.46\ \text{N}$$

$$\sigma_P = \sigma_W\left(\frac{(1-\mu_s^2)\tan\theta + 2\mu_s}{1-2\mu_s\tan\theta - \mu_s^2}\right) = 0.673$$

Thus, we have $P \sim N(13.46, 0.673^2)$ N.

Since the actual random force P_s follows $\sim N(15, 1^2)$ N, we construct $Y = P_s - P$. Therefore, the probability that the block will be lifted is $P(Y > 0)$:

$$P(Y > 0) = 1 - P(Y \le 0) = 1 - \Phi(\frac{-\mu_Y}{\sigma_Y}) = 89.87\%$$ **Ans**.