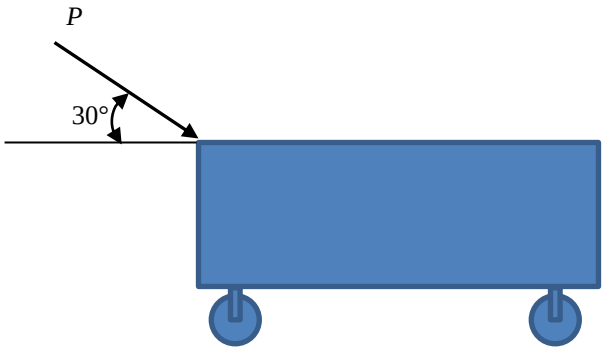
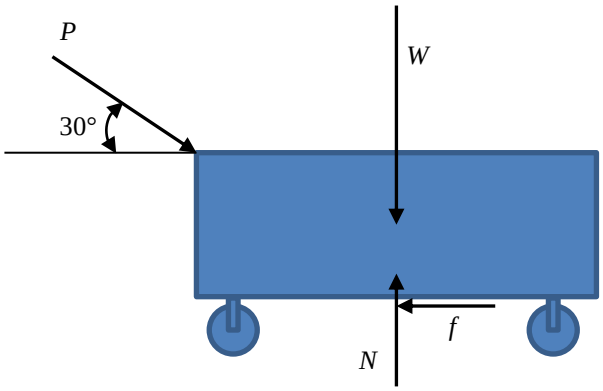
14. A random force $P \sim N(40,5^2)$ lb is applied to push a cart with wheels with a radius of $r = 2.5$ in. The weight of the cart is also a random variable $W \sim N(1000,20^2)$ lb and is independent of P. Given the coefficient of rolling resistance $a = 0.05$ in, determine the probability that the cart will start to move.



Solution



$$\sum F_y = 0; \quad N - W - P\sin 30^\circ = 0$$

Then, we have

$$N = W + P\sin 30^\circ$$

Also, the rolling resistance is

$$f = \frac{Na}{r} = \frac{(W + P\sin 30^\circ)a}{r}$$

The probability that the force P can overcome the resistance is $P(Y > 0)$, where

$$Y = P\cos 30^\circ - f = -\frac{Wa}{r} + P(\cos 30^\circ - \frac{a\sin 30^\circ}{r})$$

Therefore, we can obtain

$$\mu_Y = -\frac{\mu_W a}{r} + \mu_P(\cos 30^\circ - \frac{a\sin 30^\circ}{r}) = 14.24 \text{ lb}$$

$$\sigma_Y = \sqrt{\left(\frac{\sigma_W a}{r}\right)^2 + \left(\sigma_P(\cos 30^\circ - \frac{a\sin 30^\circ}{r})\right)^2} = 4.22$$

Finally, the probability that the force P can pull the cart is

$$P(Y > 0) = 1 - P(Y \le 0) = 1 - \Phi(\frac{-\mu_Y}{\sigma_Y}) = 99.96\%$$ **Ans**.