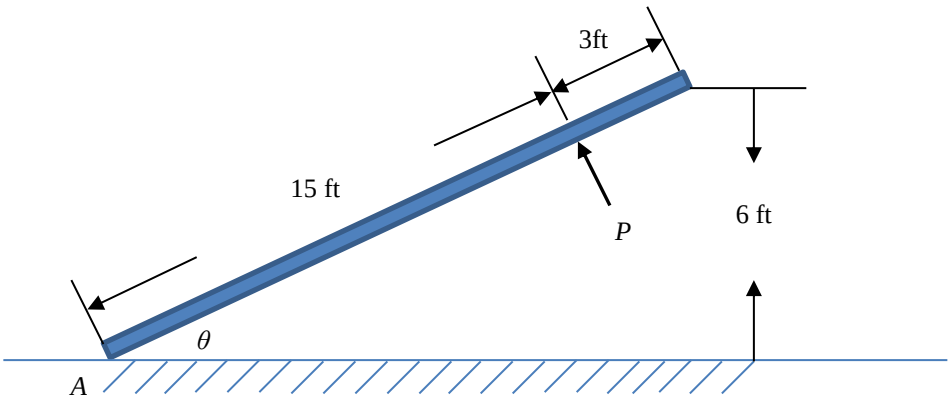
15. A normally distributed force $P \sim N(30, 2^2)$ N is applied perpendicularly to a beam which weights $m \sim N(5, 0.2^2)$ kg, where P and m are independent. Determine the probability of slipping if the coefficient of static friction between the beam and the ground at A is $\mu_s = 0.4$.



Solution

$$\theta = \arcsin(6/18) = 19.47^\circ$$

According to the force equilibrium, we have

$$\sum F_y = 0; \quad N_A + P\cos\theta - mg = 0$$

Then, we have

$$N_A = mg - P\cos\theta$$

Where, the frictional force is

$$f = \mu N_A = \mu(mg - P\cos\theta)$$

The probability of slipping is that $Y > 0$, where

$$Y = P\sin\theta - f = P\sin\theta - \mu(mg - P\cos\theta)$$

Thus, we obtain

$$\mu_Y = \mu_P \sin\theta - \mu\mu_m g + \mu\mu_P \cos\theta = 1.694 \text{ N}$$

$$\sigma_Y = \sqrt{(\sigma_P \sin\theta)^2 + (\mu\sigma_m g)^2 + (\mu\sigma_P \cos\theta)^2} = 1.276$$

Finally, the probability of slipping is

$$P(Y>0)=1-P(Y\le 0)=1-\Phi(\frac{-\mu_Y}{\sigma_Y})=90.77\%$$ **Ans**.