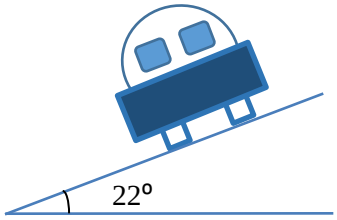
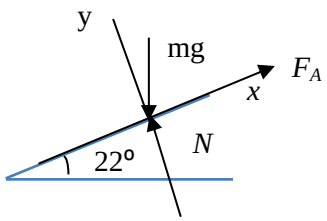
16. A car travels at a constant velocity along a straight and banked road. The tires are perpendicular to the road. The maximum static friction force follows a normal distribution $F \sim N(1250, 20^2)$ N. The car and its passengers have a mass of $m \sim N(320, 5^2)$ kg. Find the probability that the car will slip. The coefficient of static friction is $\mu_A = 0.5$ between the car and road. F and m are independent.



Solution



$$\Sigma F_x = 0;\ F_A - m(9.81)\sin 20^\circ = 0$$

$$\mu_{F_A} = \mu_m (9.81)\sin 20^\circ = 1176 \text{ N}$$

$$\sigma_{F_A} = \sigma_m (9.81)\sin 20^\circ = 18.37 \text{ N}$$

We construct the function

$$Y = F - F_A$$

$$\mu_Y = \mu_F - \mu_{F_A} = 74 \text{ N}$$

$$\sigma_Y = \sqrt{\sigma_F^2 + \sigma_{F_A}^2} = 27.16 \text{ N}$$

The probability that the car will slip is

$$p_f = \Pr(Y < 0) = \Phi(-\mu_Y / \sigma_Y) = 0.0032 \qquad \textbf{Ans.}$$