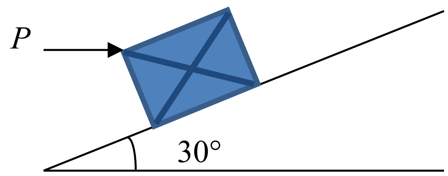
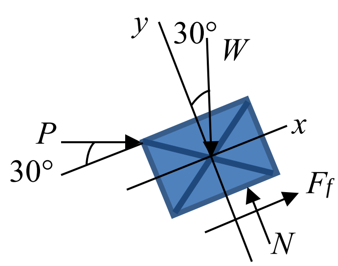
17. A horizontal force $P \sim N(100, 5^2)$ lb acts on a crate whose weight is $W \sim N(320, 10^2)$ lb. The coefficient of static friction is $\mu_s = 0.3$. Determine the probability that the crate will slip. P and W are distributed independently.



Solution



Assume no slipping

$$\Sigma F_x = 0;\ P\cos 30^\circ - W\sin 30^\circ + F_f = 0$$

$$\Sigma F_y = 0;\ N - W\cos 30^\circ = 0$$

From above two equations, we have

$$F_f = W\sin 30^\circ - P\cos 30^\circ$$

$$\mu_{F_f} = \mu_W \sin 30^\circ - \mu_P \cos 30^\circ = 73.4 \text{ lb}$$

$$\sigma_{F_f} = \sqrt{(\sigma_P \cos 30^\circ)^2 + (\sigma_W \sin 30^\circ)^2} = 6.6 \text{ lb}$$

$$N = W\cos 30^\circ$$

The maximum frictional force is

$$F_{f\_\max} = \mu_s N$$

$$\mu_{F_{f\_\max}} = \mu_s \mu_W \cos 30^\circ = 83.14 \text{ lb}$$

$$\sigma_{F_{f\_max}} = \mu_s \sigma_W \cos 30^\circ = 2.6 \text{ lb}$$

Thus, the distribution of the frictional force is $F_f \sim N(73.4, 6.6^2)$ lb and the distribution of the maximum frictional force is $F_{f\_max} \sim N(83.14, 2.6^2)$ lb.

Then we construct function $Y = F_{f\_max} - F_f$,

$$\mu_Y = \mu_{F_{f\_max}} - \mu_{F_f} = 9.74 \text{ lb}$$

$$\sigma_Y = \sqrt{\sigma_{F_{f\_max}}^2 + \sigma_{F_f}^2} = 7.11 \text{ lb}$$

Thus, the probability that the crate will slip is

$$p_f = \Pr(Y < 0) = \Pr(-\mu_Y / \sigma_Y) = 0.0852 \quad \textbf{Ans.}$$