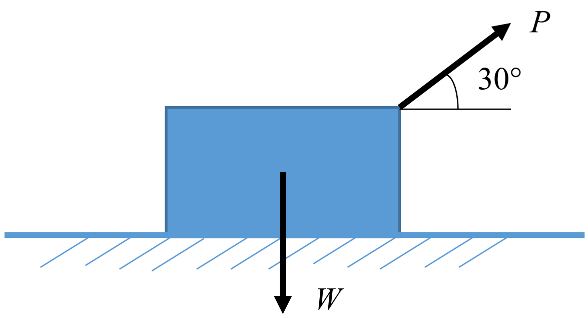
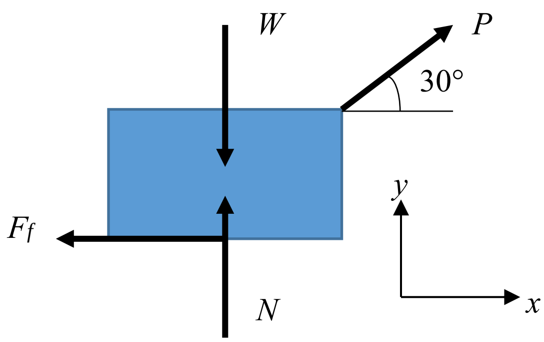
18. A force $P \sim N(100, 5^2)$ lb acts on a crate whose weight is $W \sim N(320, 10^2)$ lb. The coefficient of static friction between the crate and floor is $\mu_s = 0.3$. Determine the probability that the crate will move forward. P and W are distributed independently.



Solution



Assume no movement.

$$\Sigma F_x = 0 \quad P\cos 30^\circ - F_f = 0$$

$$\Sigma F_y = 0 \quad N + P\sin 30^\circ - W = 0$$

From above two equations, we have

$$F_f = P\cos 30^\circ$$

$$\mu_{F_f} = \mu_P \cos 30^\circ = 86.6 \text{ lb}$$

$$\sigma_{F_f} = \sigma_P \cos 30^\circ = 4.33 \text{ lb}$$

$$N = W - P\sin 30^\circ$$

The maximum frictional force is

$$F_{f\_\max} = \mu_s N = \mu_s \left(W - P\sin 30^\circ\right)$$

$$\mu_{F_{f\_\max}} = \mu_s \left(\mu_W - \mu_P \sin 30^\circ\right) = 81 \text{ lb}$$

$$\sigma_{F_{f\_\max}} = \mu_s \sqrt{\sigma_W^2 + \left(\sigma_P \sin 30^\circ\right)^2} = 3.1 \text{ lb}$$

Thus, the distribution of the frictional force is $F_f \sim N(86.6, 4.33^2)$ lb and the distribution of the maximum frictional force is $F_{f\_\max} \sim N(81, 3.1^2)$ lb.

Then we construct function $Y = F_{f\_\max} - F_f$

$$\mu_Y = \mu_{F_{f\_\max}} - \mu_{F_f} = -5.6 \text{ lb}$$

$$\sigma_Y = \sqrt{\sigma_{F_{f\_\max}}^2 + \sigma_{F_f}^2} = 5.32 \text{ lb}$$

Thus, the probability that the crate will move forward is

$$p_f = \Pr(Y < 0) = \Pr(-\mu_Y / \sigma_Y) = 0.8538 \quad \textbf{Ans.}$$