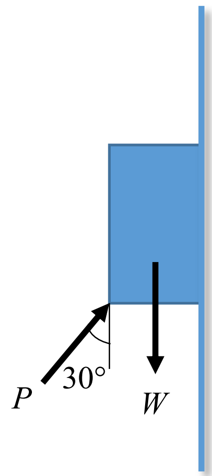
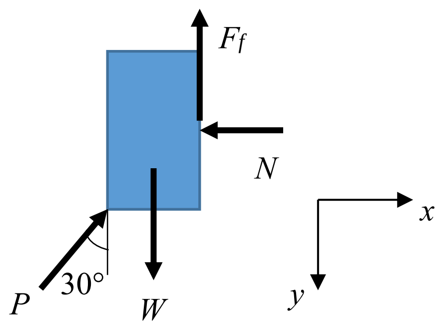
19. A crate with weight $W \sim N(320, 10^2)$ N is pushed against the wall by a force $P \sim N(350, 30^2)$ N. The coefficient of static friction between the crate and wall is $\mu_s = 0.3$. Determine the probability that the crate will slip. P and W are distributed independently.



Solution



Assume no movement.

$$\Sigma F_x = 0 \quad P\sin 30^\circ - N = 0$$

$$\Sigma F_y = 0 \quad W - P\cos 30^\circ - F_f = 0$$

From above two equations, we have

$$N = P\sin 30^\circ$$

$$F_{f\_\max} = \mu_s P \sin 30^\circ$$

$$\mu_{F_{f\_\max}} = \mu_s \mu_P \sin 30^\circ = 52.5 \text{ N}$$

$$\sigma_{F_{f\_\max}} = \mu_s \sigma_P \sin 30^\circ = 4.5 \text{ N}$$

$$F_f = W - P\cos 30^\circ$$

$$\mu_{F_f} = \mu_W - \mu_P \cos 30^\circ = 16.9 \text{ N}$$

$$\sigma_{F_f} = \sqrt{\sigma_W^2 + \left(\sigma_P \cos 30^\circ\right)^2} = 11.1 \text{ N}$$

Thus, the distribution of the frictional force is $F_f \sim N(16.9, 11.1^2)$ N and the distribution of the maximum frictional force is $F_{f\_\max} \sim N(52.5, 4.5^2)$ N.

Then we construct function $Y = F_{f\_\max} - F_f$

$$\mu_Y = \mu_{F_{f\_\max}} - \mu_{F_f} = 35.6 \text{ N}$$

$$\sigma_Y = \sqrt{\sigma_{F_{f\_\max}}^2 + \sigma_{F_f}^2} = 12 \text{ N}$$

Thus, the probability that the crate will slip is

$$p_f = \Pr(Y < 0) = \Pr(-\mu_Y / \sigma_Y) = 0.0014 \quad \textbf{Ans.}$$