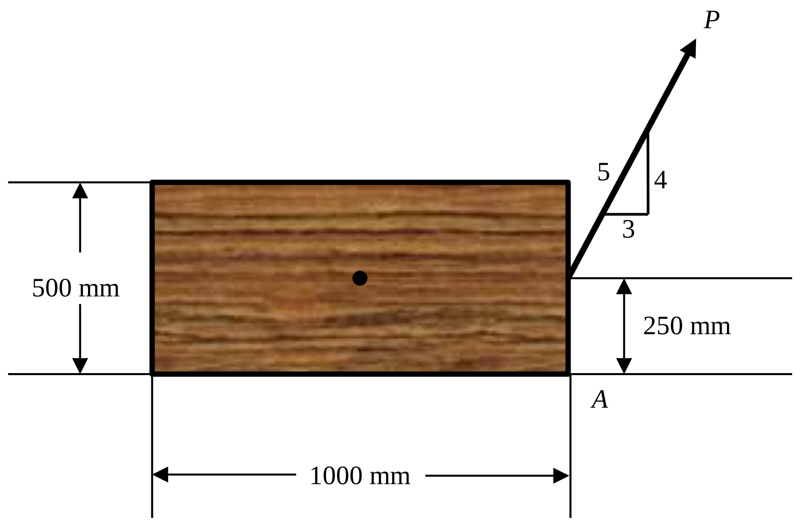
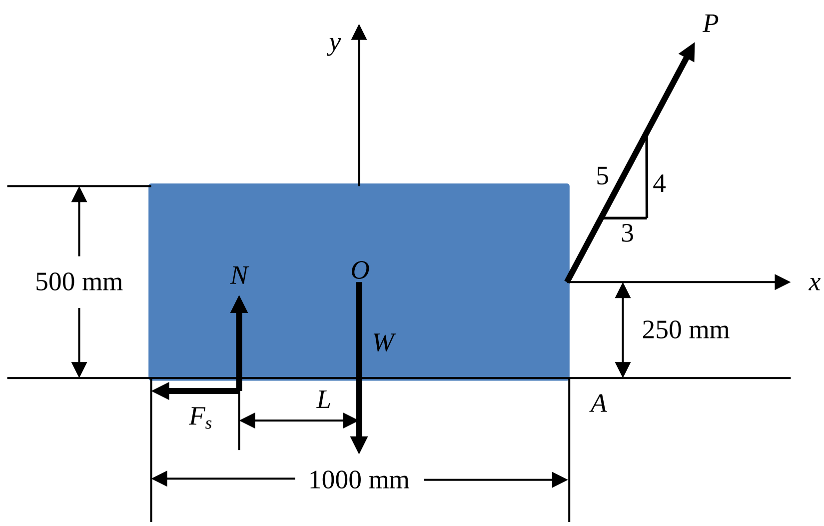
3. A random force P is applied to tow a crate of $W \sim N(50,\ 1.5^2)$ kg. P is normally distributed with a standard deviation σ_P=5. P and W are independent. What is the minimum mean of P so that the probability of towing the crate is greater than 99.99%?



Solution



$$\sum F_x = 0; \quad \frac{3}{5}P - 0.25N = 0,$$

$$\sum F_y = 0; \quad N - W(9.81) + \frac{4}{5}P = 0.$$

Solving the system of equations, with $W \sim N(50,\ 1.5^2)$ kg, we can obtain the distributions of P and N

$$\mu_N = 7.36\mu_W = 368 \text{ N},$$
$$\sigma_N = 7.36\sigma_W = 11.0,$$
$$\mu_P = 0.417\mu_N = 153.5 \text{ N},$$
$$\sigma_P = 0.417\sigma_N = 4.6.$$

Thus, we have obtained the distributions of the normal force and the external force P: $N \sim N(368, 11^2)$ N and $S \sim N(153.5, 4.6^2)$ N. **Ans.**

Thus, we can construct

$$Y = P - S.$$

And

$$\mu_Y = \mu_P - 153.5,$$
$$\sigma_Y = \sqrt{\sigma^2{}_Y + \sigma^2{}_S} = 6.794.$$

The probability of towing the crate can be calculated as

$$P(Y \geq 0) = 1 - P(Y < 0) = 1 - \Phi(-\frac{\mu_P - 153.5}{6.794}) = 99.99\%.$$

Solve the above equation inversely, we can obtain the minimum mean μ_P=179 N. **Ans.**