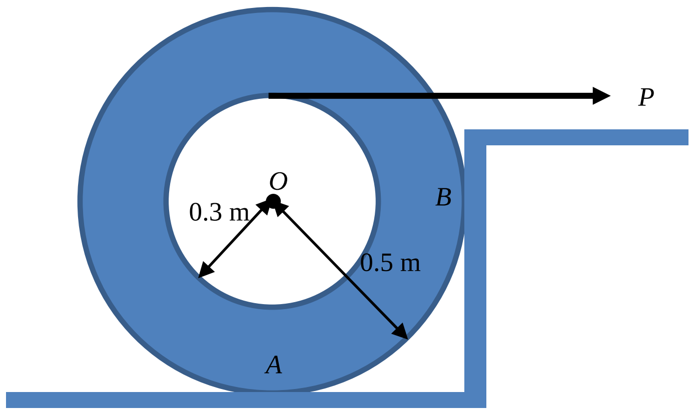
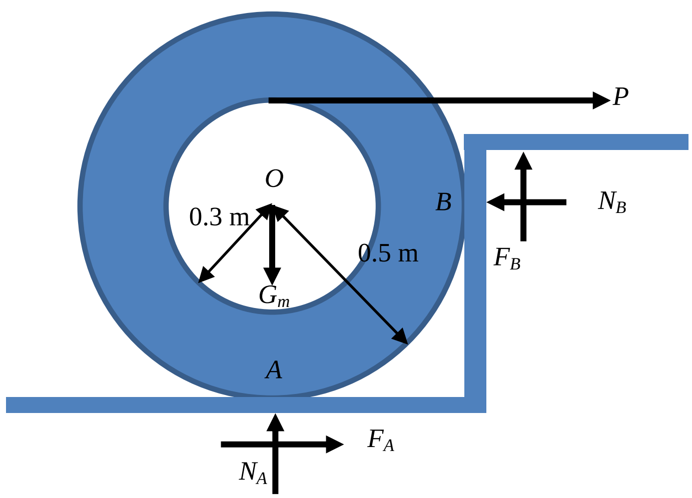
4. The spool rests on the ground at A and against the wall at B. The weight of the spool follows a distribution $m \sim N(130, 4.5^2)$ kg due to the manufacturing uncertainty. (1) Determine the maximum force P so that the probability of no motion is less than 0.01%. (2) If the strength of wire, which is independent of m, exerted on the spool follows another normal distribution $T \sim N(700, 6^2)$ N, determine the probability that the wire may break when force P begins pulling the wire horizontally off the spool. The coefficient of static friction between the spool and its position of contact is μ_s=0.2.



Solution



$$\sum F_y = 0; \quad N_A + F_B - G_m = 0,$$
$$\sum F_x = 0; \quad F_A - N_B + P = 0,$$
$$\sum M_B = 0; \quad -P(0.3) + G_m(0.5) - N_A(0.5) + F_A(0.5) = 0$$

Where $F_A = \mu_s N_A = 0.2 N_A$ and $F_B = \mu_s N_B = 0.2 N_B$.

Solving the above equations, with $m \sim N(130, 4.5^2)$ kg, we can obtain

$$P = \frac{1.5}{2.9} G$$

$$\mu_P = \frac{1.5}{2.9} \mu_m (9.81) = 659.64 \text{ N}$$
$$\sigma_P = \frac{1.5}{2.9} \sigma_m (9.81) = 22.83$$

Thus, we have the distribution: $P \sim N(659.64, 22.83^2)$ N.

Therefore, the maximum magnitude P so that the probability of no motion is less than 0.01% can be computed by the underneath equation inversely

$$P\left(P < P_{\max}\right) = \Phi(\frac{P_{\max} - 659.64)}{22.83}) = 99.99\%.$$

Finally, we have $P_{\max}$=744 N. **Ans.**

(2) The distribution of the wire strength is $T \sim N(700, 6^2)$ N. Therefore, we could have

$$Y = T - P$$
$$\mu_Y = \mu_T - \mu_P = 40.36 \text{ N}$$
$$\sigma_Y = \sqrt{\mu_T^2 + \mu_P^2} = 23.61 \text{ N}$$

The probability that the wire may break is

$$P(Y < 0) = \Phi(-\frac{40.36}{23.61}) = 0.0437$$ **Ans.**