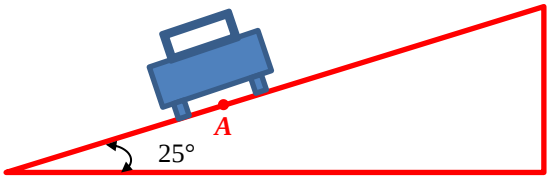
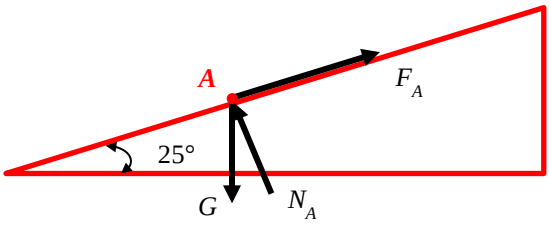
5. A car travels along a banked road with a constant speed, the mass of the car is $m \sim N(700,5^2)$ kg, and the coefficient of static friction μ_s =0.48. What is the probability that the car will slip?



Solution



$$\sum F_y = 0; \quad N_A - m(9.81)\cos 25^\circ = 0,$$
$$\sum F_x = 0; \quad F_A - m(9.81)\sin 25^\circ = 0,$$
$$F_{A\max} = \mu_s N_A.$$

Since m follows normal distributions with $m \sim N(700,5^2)$ kg and μ_s =0.48, we have

$$\mu_{N_A} = (9.81)\cos 25^\circ \mu_m = 6223.6 \text{ N},$$
$$\sigma_{N_A} = (9.81)\cos 25^\circ \sigma_m = 44.5,$$
$$\mu_{F_A} = (9.81)\sin 25^\circ \mu_M = 2902.1 \text{ N},$$
$$\sigma_{F_A} = (9.81)\sin 25^\circ \sigma_m = 20.7,$$
$$\mu_{F_{A\max}} = 0.48\mu_{N_A} = 2987.3 \text{ N},$$
$$\sigma_{F_{A\max}} = 0.48\sigma_{N_A} = 21.3.$$

Thus, the distribution of friction force at A is: $F_A \sim N(2902.1, 20.7)$ N.

We construct function Y as

$$Y = F_{A\max} - F_A$$

Thus

$$\mu_Y = \mu_{F_{A\max}} - \mu_{F_A} = 85.2 \text{ N},$$
$$\sigma_Y = \sqrt{\sigma^2{}_{F_{A\max}} + \sigma^2{}_{F_A}} = 29.7.$$

The probability that the motorcycle might fall is

$$P(Y < 0) = \Phi(-\frac{85.2}{29.7}) = 0.0021.$$ **Ans.**