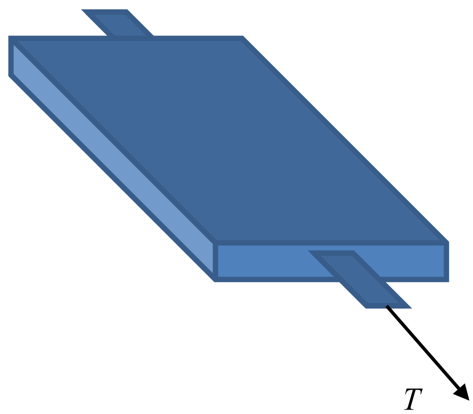
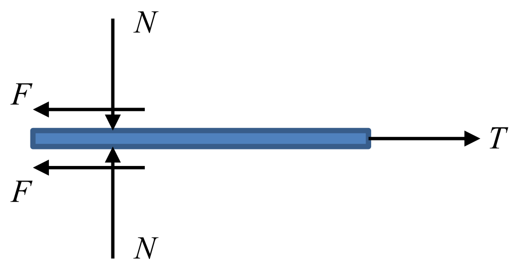
6. A random force $T \sim N(0.95, 0.015^2)$ lb is applied to pull a bookmark with a width of 1 in. The weight of the book is $W \sim N(12, 0.15^2)$ lb, and the coefficient of static friction between the bookmark and the paper is μ_s=0.6. If the pages are 8 in. by 10 in., determine the probability that the bookmark will start to move out. Assume the pressure on each page and the bookmark is uniform.



Solution



Pressure on book mark is:

$$P = \frac{W}{2A}$$

Then, with $W \sim N(12, 0.15^2)$ lb, we could have the distribution of pressure P:

$$\mu_P = \frac{\mu_W}{2A} = \frac{12}{2(8)(10)} = 0.075 \text{ lb / in}^2$$

$$\sigma_P = \frac{\sigma_W}{2A} = \frac{0.15}{2(8)(10)} = 9.375 \times 10^{-4}$$

Normal force on bookmark:

$$N = PA_{\text{mark}}$$

$$A_{\text{mark}} = 1(10) = 10 \text{ in}^2$$

Then, we have:

$$\mu_N = \mu_P A_{\text{mark}} = 0.75 \text{ lb}$$

$$\sigma_N = \sigma_P A_{\text{mark}} = 9.375 \times 10^{-3}$$

Thus, we can obtain the distribution of friction force F with $\mu_s = 0.6$:

$$\mu_F = \mu_N (0.6) = 0.45 \text{ lb}$$

$$\sigma_F = \sigma_N (0.6) = 5.625 \times 10^{-3}$$

Finally, we can calculate the distribution of the drag force T_s which is required to pull the bookmark out:

$$\sum F_x = 0; \quad T_s - 2F = 0$$

$$\mu_{T_s} = \mu_F (2) = 0.9 \text{ lb}$$

$$\sigma_{T_s} = \sigma_F (2) = 0.01125$$

Then, we can construct

$$Y = T - T_s$$

$$\mu_Y = \mu_T - \mu_{T_s} = 0.05 \text{ lb}$$

$$\sigma_Y = \sqrt{\sigma_T^2 + \sigma_{T_s}^2} = 0.01875$$

Finally the probability the bookmark can be moved out is

$$P(Y \geq 0) = 1 - P(Y < 0) = 1 - \Phi\left(\frac{-0.05}{0.01875}\right) = 0.9962$$ **Ans.**