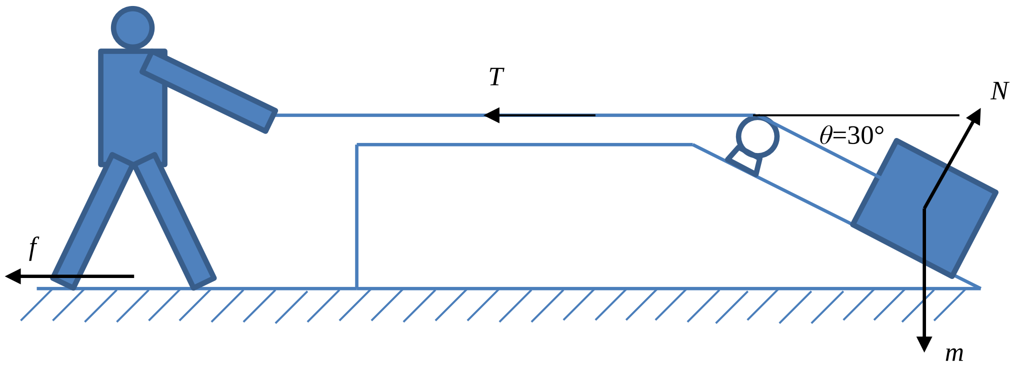
7. A man is pulling a crate on a frictionless slant. If the weight of the crate follows a normal distribution $m \sim N(90, 2^2)$ kg. (1) Determine the distribution of the minimum force T for the man to pull the crate. (2) If the man has a weight of 75kg and the static friction between him and the floor follows another normal distribution $\mu_s \sim N(0.65, 0.01^2)$, which is independent from m, determine the probability that man may fail to pull the crate if he uses the minimum force T.



Solution

(1)

On crate, we have:

$$T = mg\sin\theta$$

Thus, we can obtain:

$$\mu_T = \mu_m g\sin\theta = 441.45 \text{ N}$$

$$\sigma_T = \sigma_m g\sin\theta = 9.81$$

$$T \sim N(441.45, 9.81^2)\ N$$ **Ans.**

(2)

The maximum friction on A is:

$$f = \mu_s N$$

Then, we can have:

$$\mu_f = \mu_{\mu_s}(75)(9.81) = 478.24 \text{ N}$$

$$\sigma_f = \sigma_{\mu_s}(75)(9.81) = 7.36$$

The man may fail to pull the crate is $T > f$ or $Y > 0$, where

$$Y = T - f$$

Hereby, we can obtain:

$$\mu_Y = \mu_T - \mu_f = -36.79 \text{ N}$$

$$\sigma_Y = \sqrt{\sigma^2{}_T + \sigma^2{}_f} = 12.26$$

Consequently, the probability is:

$$P(Y > 0) = 1 - P(Y \leq 0) = 1 - \Phi(\frac{36.79}{12.26}) = 0.0013$$ **Ans**.