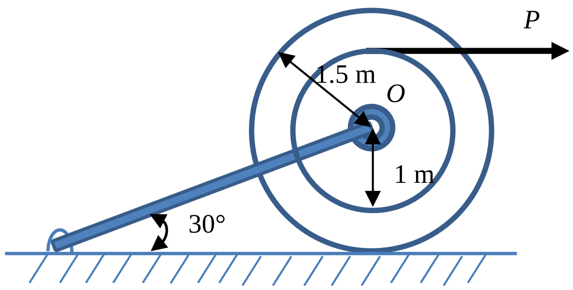
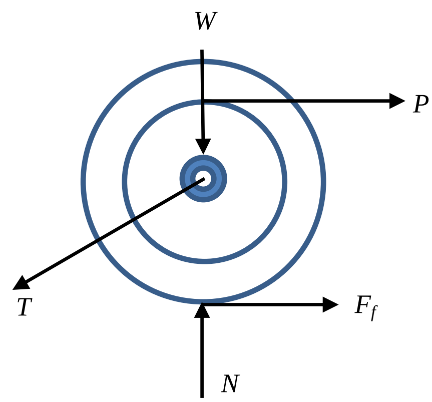
8. A force P which follows a normal distribution $P \sim N(2, 0.025^2)$ lb is applied to the spool as shown below. Determine the probability that the spool will slide if the weight of the spool follows another independent normal distribution $W \sim N(1, 0.05^2)$ lb and the static friction between the ground and the spool is μ_s=0.42.



Solution



$$\sum M_O = 0 \quad P(1) = F_f(1.5)$$

Then, we can obtain:

$$\mu_{F_f} = \frac{2}{3}\mu_P = 1.33\,\text{lb}$$

$$\sigma_{F_f} = \frac{2}{3}\sigma_P = 0.0167$$

$$F_f \sim N(1.33, 0.0167^2) \text{ lb.}$$

Also, we could have:

$$\sum F_x = 0 \quad P + F_f = T\cos 30^\circ$$

$$\mu_T = \frac{2}{\sqrt{3}}(\mu_P + \mu_{F_f}) = 3.849 \text{ lb}$$

$$\sigma_T = \frac{2}{\sqrt{3}}\sqrt{\sigma_P^2 + \sigma_{F_f}^2} = 0.0347$$

$$T \sim N(3.849, 0.0347^2) \text{ lb.}$$

Also, we have:

$$\sum F_y = 0 \quad N = W + T\sin 30^\circ$$

And:

$$f_{\max} = 0.42N$$

Thus, we can obtain:

$$\mu_{f_{\max}} = 0.42(\mu_W + \mu_T \sin 30^\circ) = 1.2283 \text{ lb}$$

$$\sigma_{f_{\max}} = 0.42\sqrt{\sigma_W^2 + (\sigma_T \sin 30^\circ)^2} = 0.0222$$

$$f_{\max} \sim N(1.2283, 0.0222^2) \text{ lb.}$$

The probability that the spool will slide is that $F_f \geq f_{\max}$ or $Y \geq 0$, where,

$$Y = F_f - f_{\max}$$

Then, we can obtain:

$$\mu_Y = \mu_{F_f} - \mu_{f_{\max}} = 0.105 \text{ lb}$$

$$\sigma_Y = \sqrt{\sigma_{F_f}^2 + \sigma_{f_{\max}}^2} = 0.0278$$

Therefore, the probability that the spool will slide is:

$$P(Y > 0) = 1 - P(Y \leq 0) = 1 - \Phi(\frac{-0.105}{0.0278}) = 0.9999$$ **Ans**.