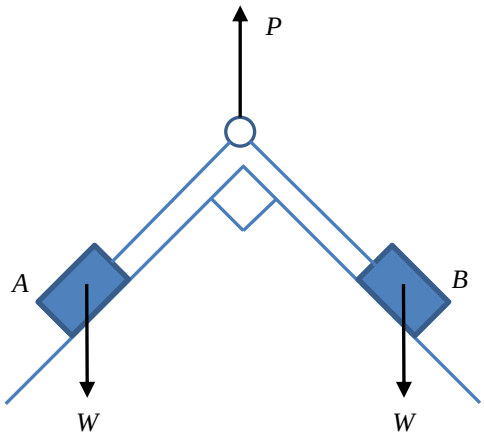
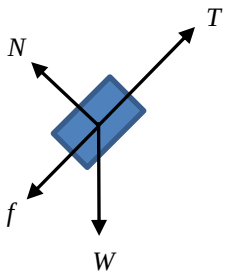
9. Blocks *A* and *B* are connected by two links as shown. The force *P* and coefficient of friction are independently and normally distributed with $P \sim N(75,\ 2^2)$ N and $\mu_s \sim N(0.5,\ 0.002^2)$, respectively. Determine the probability that two blocks will start to move if *A* and *B* both have mass of 5 kg.





Solution

$$N = mg\cos 45^\circ = 5(9.81)\cos 45^\circ = 34.68 \text{ N}$$

And

$$f = \mu_s N$$

Then, we have

$$\mu_f = \mu_{\mu_s} N = 17.34 \text{ N}$$

$$\sigma_f = \sigma_{\mu_s} N = 0.0694$$

$$f \sim N(17.34,\ 0.0694^2) \text{ N}.$$

Also, we have

$$T = f + mg\sin 45^\circ$$

Therefore, we can obtain

$$\mu_T = \mu_f + mg\sin 45^\circ = 52.02 \text{ N}$$

$$\sigma_T = \sigma_f = 0.0694$$

$$T \sim N(52.02,\ 0.0694^2)\ \text{N}$$

The probability that two blocks start to move is $P \geq 2T\cos 45°$ or $Y \geq 0$, where

$$Y = P - 2T\cos 45^\circ$$

$$\mu_Y = \mu_P - 2\cos 45^\circ \mu_T = 1.425$$

$$\sigma_Y = \sqrt{\sigma_P^2 + (2\cos 45^\circ \sigma_T)^2} = 2$$

$$Y \sim N(1.425,\ 2^2)\ \text{N}$$

Consequently, the probability that two blocks will start to move is

$$P(Y > 0) = 1 - P(Y \leq 0) = 1 - \Phi(\frac{-1.425}{2}) = 76.17\%$$ **Ans**.