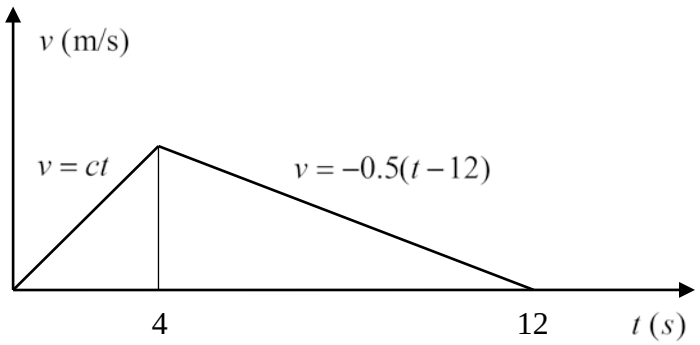
1-2. A car travels along a straight road with the speed shown in the $v-t$ graph. Assume c is greater than zero and follows a normal distribution $c \sim N(1, 0.1^2)$, plot the $a-t$ graph and calculate the probability that the speed v reaches 5 m/s.



Solution

$a-t$ graph: for $0 \le t \le 4\ s$

$$v = ct$$

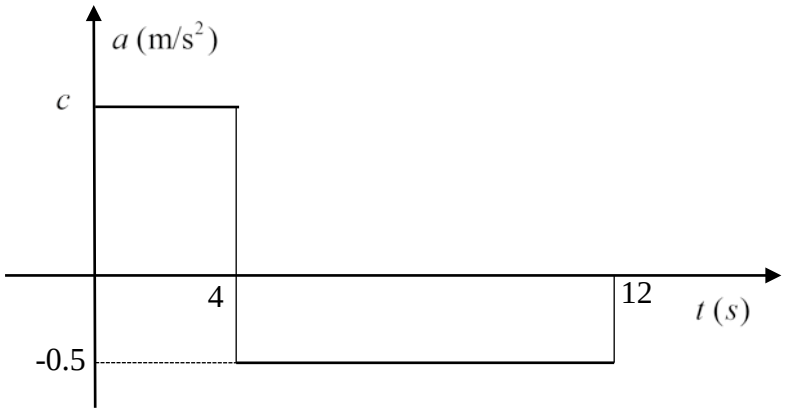
$$a = \frac{dv}{dt} = c$$

For $4 < t \le 12\ s$

$$v = -0.5(t-12)$$

$$a = \frac{dv}{dt} = -0.5$$

The graph of the acceleration is shown below.



Since $c \sim N(1, 0.1^2)$ and $v = ct$, when $t = 4$, the speed reaches its maximum, therefore,

$$P(v > 5) = P(ct > 5) = P(Y = 5 - ct < 0)$$

And

$$\mu_Y = 5 - \mu_c t = 5 - 1(4) = 1 \text{ m/s}$$

$$\sigma_Y = t\sigma_c = 4(0.1) = 0.4 \text{ m/s}$$

So

$$\begin{aligned} P(v > 5) &= P(Y < 0) \\ &= \Phi\left(\frac{0 - \mu_Y}{\sigma_Y}\right) \\ &= 0.0062 \end{aligned}$$

Ans.