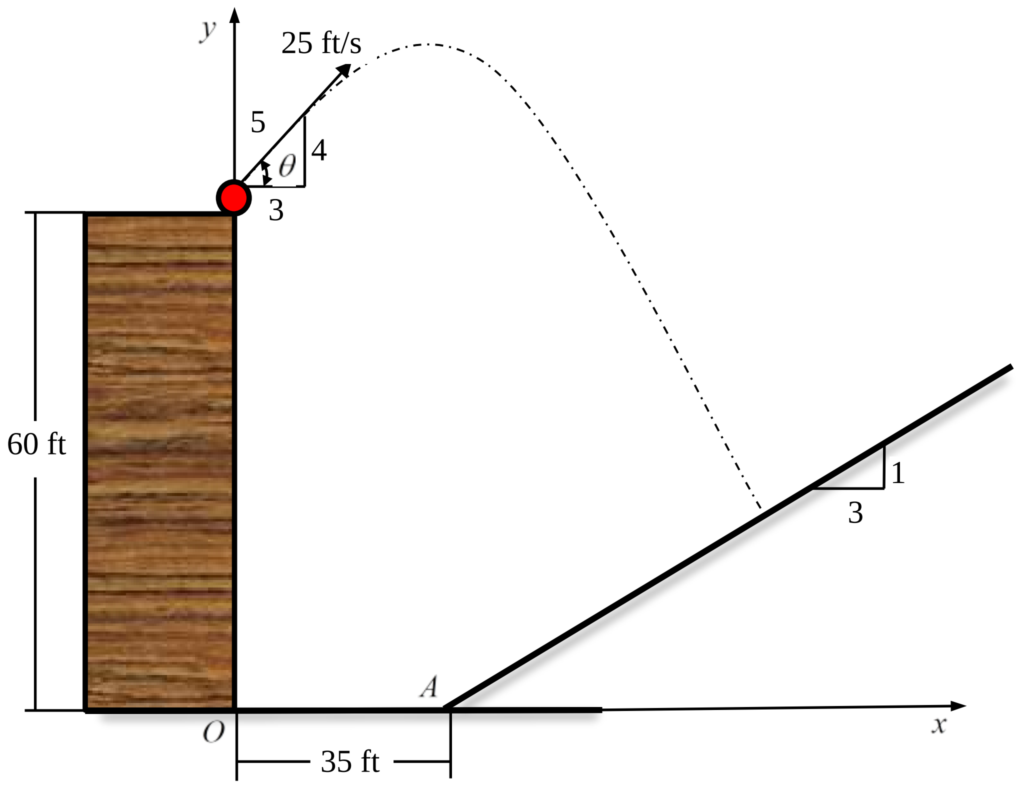
1-3. A ball is thrown from a tower shown in the figure. If the initial velocity is 25 ft/s.

(1) Determine the x and y coordinates where the ball strikes the slope. Assume $x_A = 35\text{ ft}$.

(2) If x_A is a random variable and follows a normal distribution $x_A \sim N(35, 3^2)\text{ ft}$, calculate the probability that the ball will not hit the slope.



Solution

(1) Assume the ball hits the slope.

Since $s = s_0 + v_0 t$, we have

$$x = x_0 + v_0 \cos\theta t$$
$$= 0 + 25\left(\frac{3}{5}\right)t = 15t$$

$$y = y_0 + v_0 \sin\theta t + \frac{1}{2}at^2$$
$$= 60 + 25\left(\frac{4}{5}\right)t + \frac{1}{2}(-32.2)t^2 = 60 + 20t - 16.1t^2$$

The equation of the slope is $y - y_A = k(x - x_A)$

where $(x_A, y_A) = (35, 0)$ ft and $k = 1/3$, and therefore

$$y = \frac{1}{3}(x - 35)$$

$$60 + 20t - 16.1t^2 = \frac{1}{3}(15t - 35)$$

Solving the equations above and choosing the positive root yields

$$t = 2.6265\ \text{s}$$

thus

$$x = 15t = 15(2.6265) = 39.397\ \text{ft}$$

$$y = \frac{1}{3}(x - 35) = \frac{1}{3}(39.397 - 35) = 1.466\ \text{ft}$$

Therefore, the coordinates are $(x, y) = (39.397, 1.466)$ ft . **Ans.**

(2) When $x_A \sim N(35, 3^2)$ ft , the probability that the ball will not hit the slope

$$P(x < x_A) = 1 - P(x_A < x) = 1 - \Phi\left(\frac{x - \mu_{x_A}}{\sigma_{x_A}}\right)$$

$$= 1 - \Phi\left(\frac{39.379 - 35}{3}\right)$$

$$= 0.0714$$

Therefore, the probability that the ball will not hit the slope is 0.0714. **Ans.**