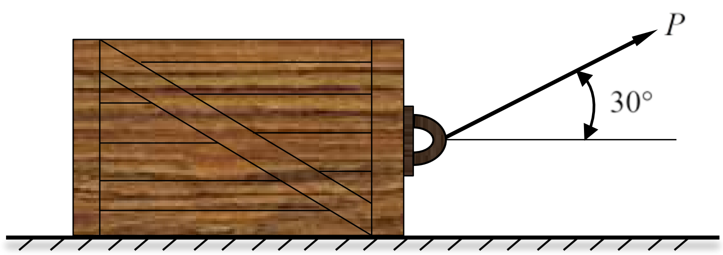
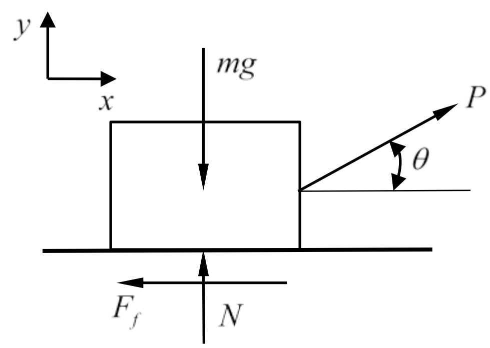
2-1. The initial velocity of the block is $v_0 = 3\,\text{m/s}$, and the initial position of the block is $s_0 = 5\,\text{m}$. If a normally distributed force $P \sim N(300, 20^2)\,\text{N}$ is applied to the rope and the coefficient of kinetic friction between the 100-kg block and the ground is $\mu_k = 0.2$. Determine the means and standard deviations of the position and velocity of the block when $t = 3\,\text{s}$.



Solution



Referring to the free-body diagram of the block, we have

$$\Sigma F_y = 0$$
$$P\sin\theta + N - mg = 0$$
$$N = mg - P\sin\theta$$

$$\Sigma F_x = ma_x$$
$$P\cos\theta - \mu_k N = ma$$
$$a = \frac{1}{m}\left(P\cos\theta - \mu_k N\right)$$
$$= \frac{1}{m}\left[P\cos\theta - \mu_k\left(mg - P\sin\theta\right)\right]$$
$$= \frac{P}{m}\left(\cos\theta + \mu_k\sin\theta\right) - \mu_k g$$

Therefore

$$\begin{aligned} v &= v_0 + at \\ &= 3 + \left[\frac{P}{m} \left(\cos\theta + \mu_k \sin\theta \right) - \mu_k g \right] t \end{aligned}$$

$$\begin{aligned} \mu_v &= 3 + \left[\frac{\mu_P}{m} \left(\cos\theta + \mu_k \sin\theta \right) - \mu_k g \right] t \\ &= 3 + \left[\left(\frac{300}{100} \right) \left(\cos 30^\circ + 0.2 \sin 30^\circ \right) - 0.2(9.81) \right] (3) \\ &= 5.81 \text{ m/s} \end{aligned}$$

$$\sigma_v = \frac{\cos\theta t}{m} \sigma_P = \frac{\cos 30^\circ (3)}{100} \left(20 \right) = 0.52 \text{ m/s}$$

$$\begin{aligned} s &= s_0 + v_0 t + \frac{1}{2} a t^2 \\ &= 5 + 3t + \frac{1}{2} \left[\frac{P}{m} \left(\cos\theta + \mu_k \sin\theta \right) - \mu_k g \right] t^2 \end{aligned}$$

$$\begin{aligned} \mu_s &= 5 + 3t + \frac{1}{2} \left[\frac{\mu_P}{m} \left(\cos\theta + \mu_k \sin\theta \right) - \mu_k g \right] t^2 \\ &= 5 + 3(3) + \frac{1}{2} \left[\left(\frac{300}{100} \right) \left(\cos 30^\circ + 0.2 \sin 30^\circ \right) - 0.2(9.81) \right] (3^2) \\ &= 18.21 \text{ m} \end{aligned}$$

$$\sigma_s = \frac{\cos\theta t^2}{2m} \sigma_P = \frac{\cos 30^\circ (3^2)}{2(100)} \left(20 \right) = 0.78 \text{ m}$$

Therefore, $v \sim N(5.81, 0.52^2)$ m/s and $s \sim N(18.21, 0.78^2)$ m. **Ans.**