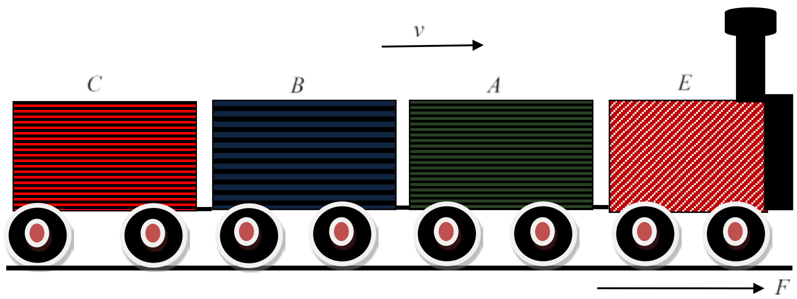
2-11. As shown in the figure, a train consists of an engine and three cars. The wheels of the engine provide a resultant frictional tractive force F, which gives the train forward motion. The wheels of the cars roll freely. The train starts from rest, and it takes $t = 30\ \text{s}$ to increase its speed uniformly to $80\ \text{km/h}$. If the mass of the engine follows a normal distribution $m_E \sim N(30, 2^2)\ \text{Mg}$, and the masses of the three cars are the same, $m_A = m_B = m_C = m_0$, which also follows a normal distribution $m_0 \sim N(80, 3^2)\ \text{Mg}$. Determine the distributions of frictional tractive force F and the force T developed at the coupling between the engine and the first car.



Solution

$$(v_x)_2 = 80\ \text{km/h} = 22.22\ \text{m/s}$$

System

$$(\underset{\rightarrow}{+})\quad m(v_x)_1 + \Sigma\int F_x dt = m(v_x)_2$$

$$0 + Ft = (m_E + m_A + m_B + m_C)(v_x)_2$$

$$F = \frac{(m_E + m_A + m_B + m_C)(v_x)_2}{t}$$

$$\mu_F = \frac{(\mu_{m_E} + \mu_{m_A} + \mu_{m_B} + \mu_{m_C})(v_x)_2}{t} = 200\ \text{kN}$$

$$\sigma_F = \sqrt{\left(\frac{(v_x)_2}{t}\right)^2 \left(\sigma_{m_E}^2 + \sigma_{m_A}^2 + \sigma_{m_B}^2 + \sigma_{m_C}^2\right)} = 4.12\ \text{kN}$$

Therefore, $F \sim N(200, 4.12^2)\ \text{kN}$. **Ans.**

Three cars

$$(\underset{\rightarrow}{+})\quad m(v_x)_1 + \Sigma\int F_x dt = m(v_x)_2$$

$$0 + Tt = (m_A + m_B + m_C)(v_x)_2$$

$$T = \frac{(m_A + m_B + m_C)(v_x)_2}{t}$$

$$\mu_T = \frac{(\mu_{m_A} + \mu_{m_B} + \mu_{m_C})(v_x)_2}{t} = 177.78 \text{ kN}$$

$$\sigma_T = \sqrt{\left(\frac{(v_x)_2}{t}\right)^2 (\sigma_{m_A}^2 + \sigma_{m_B}^2 + \sigma_{m_C}^2)} = 3.85 \text{ kN}$$

Thus $T \sim N(177.78, 3.85^2)$ kN . **Ans.**