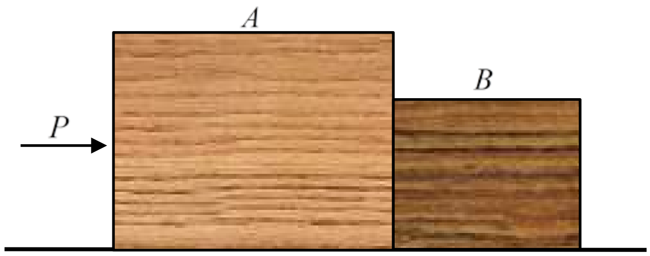
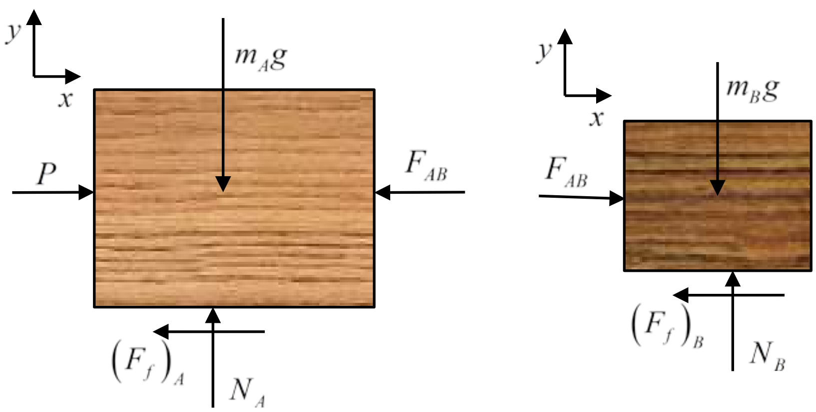
2-12. Crates A and B weigh m_A and m_B, respectively. The coefficient of kinetic friction between the ground and the crates follows a normal distribution $\mu_k \sim N(\mu_{\mu_k}, \sigma_{\mu_k}^2)$. If the crates start from rest, determine the distribution of their speed after t and the distribution of the force exerted by crate A on crate B during the motion.



Solution:

Free-Body Diagram: the free-body diagram of crates A and B are shown below.



Principle of Impulse and Momentum:

Crate A $\quad (+\uparrow)\ N_A - m_A g = 0$

The friction force acting on crate A

$$(F_f)_A = \mu_k N_A = \mu_k m_A g$$

Then $\quad m_A (v_1)_x + \Sigma \int_0^t F_x dt = m_A (v_2)_x$

$$m_A(0) + Pt - (F_f)_A t - F_{AB} t = m_A (v_2)_x$$

Therefore $$Pt - \mu_k N_A t - F_{AB} t = m_A \left(v_2\right)_x$$

Crate B

$$\left(+\uparrow\right) \; N_B - m_B g = 0$$

The friction force acting on crate B

$$\left(F_f\right)_B = \mu_k N_B = \mu_k m_B g$$

Then $$m_B \left(v_1\right)_x + \Sigma\int_0^t F_x dt = m_B \left(v_2\right)_x$$

$$m_B(0) + F_{AB} t - \left(F_f\right)_B t = m_B \left(v_2\right)_x$$

$$F_{AB} t - \mu_k N_B t = m_B \left(v_2\right)_x$$

$$(Pt - \mu_k N_A t - F_{AB} t) + (F_{AB} t - \mu_k N_B t) = m_A \left(v_2\right)_x + m_B \left(v_2\right)_x$$

Then we have

$$\left(v_2\right)_x = \frac{Pt - \mu_k t\left(N_A + N_B\right)}{m_A + m_B} = \frac{Pt}{m_A + m_B} - \mu_k g t$$

Thus,

$$\mu_{\left(v_2\right)_x} = \frac{\mu_P t}{m_A + m_B} - \mu_{\mu_k} g t$$ **Ans.**

$$\sigma_{\left(v_2\right)_x} = \sqrt{\left(\frac{t}{m_A + m_B}\right)^2 \sigma_P^2 + \left(gt\right)^2 \sigma_{\mu_k}^2}$$ **Ans.**

From $F_{AB} t - \mu_k N_B t = m_B \left(v_2\right)_x$, we have

$$F_{AB} = \frac{m_B \left(v_2\right)_x + \mu_k N_B t}{t}$$

$$= \frac{m_B}{m_A + m_B} P$$

Thus,

$$\mu_{F_{AB}} = \frac{m_B}{m_A + m_B} \mu_P$$ **Ans.**

$$\sigma_{F_{AB}} = \frac{m_B}{m_A + m_B} \sigma_P$$ **Ans.**