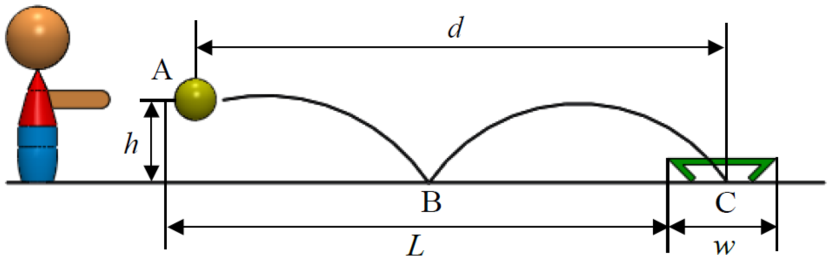
2-14. A girl throws a ball into a bucket with a horizontal velocity $v_A = 5$ m/s at point *A*, whose height is $h = 1.4$ m. The width of the bucket is $w = 0.6$ m, and the distance between the front edge of the bucket and the girl is $L = 5$ m. If the coefficient of restitution follows a normal distribution $e \sim N(0.5, 0.05^2)$, determine the likelihood that the ball reaches the target. Neglect the height of the bucket.



Solution:

From *A* to *B*

$$h = \frac{1}{2} g t_B^2$$

$$t_B = \sqrt{\frac{2h}{g}}$$

$$(v_{By})_1 = -g t_B = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}$$

At *B*:

$$e = \frac{(v_{By})_2 - 0}{0 - (v_{By})_1}$$

$$(v_{By})_2 = -e\left(v_{By}\right)_1 = e\sqrt{2gh}$$

From *B* to *C*

$$v_{Cy} = -(v_{By})_2$$

and

$$v_{Cy} = (v_{By})_2 - g t_C$$

Thus,

$$t_C = \frac{2(v_{By})_2}{g} = 2e\sqrt{\frac{2h}{g}}$$

Finding the distance the ball travels

$$d = v_A(t_B + t_C) = v_A(1+2e)\sqrt{\frac{2h}{g}}$$

Finding d mean

$$\mu_d = v_A(1+2\mu_e)\sqrt{\frac{2\text{h}}{g}} = 5\left(1+2\left(0.5\right)\right)\sqrt{\frac{2\left(1.4\right)}{9.81}} = 5.34 \text{ m}$$

$$\sigma_d = 2v_A\sqrt{\frac{2h}{g}}\sigma_e = 2\left(5\right)\sqrt{\frac{2\left(1.4\right)}{9.81}}\left(0.05\right) = 0.27 \text{ m}$$

Solving the probability

$$\begin{aligned}
\text{P}_{target} &= \Pr\left\{L \le d \le L + w\right\} = \Pr\left\{d \le L + w\right\} - \Pr\left\{d \le L\right\} \\
&= \Phi\left(\frac{L + w - \mu_d}{\sigma_d}\right) - \Phi\left(\frac{L - \mu_d}{\sigma_d}\right) \\
&= 0.73
\end{aligned}$$

Therefore, the probability that the ball gets into the bucket is 0.73**.** **Ans.**