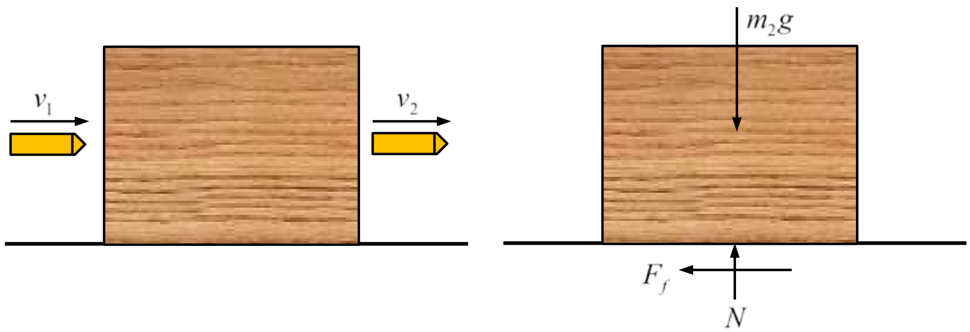
2-15. A bullet hits a block at rest at a horizontal speed v_1 and exits the block at speed v_2. Both speeds v_1 and v_2 are measured and treated as random variables due to the measurement uncertainty. They are independently distributed with $v_1 \sim N(1000, 10^2)$ m/s and $v_2 \sim N(200, 2^2)$ m/s . The mass of the bullet and the wooden block are $m_1 = 100$ g and $m_2 = 20$ kg , respectively. The coefficient of kinetic friction between the block and floor is $\mu_k = 0.3$. How long does the block slide on the floor before it comes to rest again?



Solution:

Conservation of linear momentum

$$m_1 v_1 = m_1 v_2 + m_2 v_{\mathrm{B}}$$

$$v_{\mathrm{B}} = \frac{m_1(v_1 - v_2)}{m_2}$$

After the bullet exits the block, the only force acts on the block in x-direction is the friction force, from the free-body diagram, we have

$$F_f = N\mu_k = m_1 g \mu_k$$

When the block comes to rest again, $v_3 = 0$ m/s .

Principle of impulse and momentum of the block:

$$m_2 v_B + \Sigma \int F_f dt = m_2 v_3$$

$$m_2 v_B - m_2 g \mu_k t = 0$$

$$t = \left(\frac{v_B}{g \mu_k} \right) = \frac{m_1(v_1 - v_2)}{m_2 g \mu_k}$$

$$\mu_t = \frac{m_1(\mu_{v_1} - \mu_{v_2})}{m_2 g \mu_k} = \frac{0.1(1000 - 200)}{20(9.81)(0.3)} = 1.36 \text{ s}$$

$$\sigma_t = \frac{m_1}{m_2 g \mu_k}\sqrt{\sigma_{v_1}^2 + \left(-\sigma_{v_2}^2\right)^2} = \frac{0.1}{20(9.81)(0.3)}\sqrt{10^2 + 2^2} = 0.017 \text{ s}$$

Therefore, $t \sim N(1.36, 0.017^2)$ s. **Ans.**