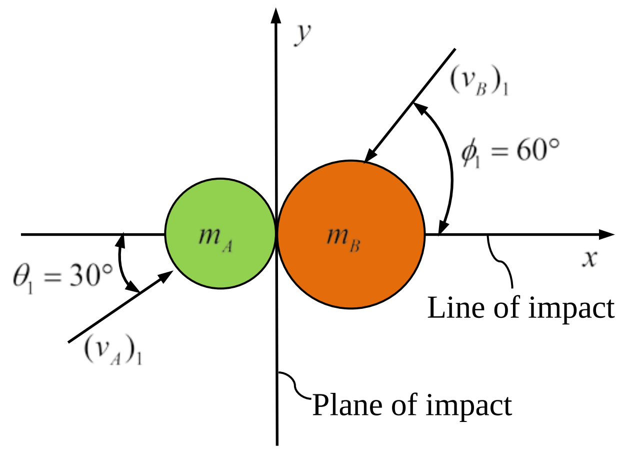
2-17. Two smooth disks A and B of 5 kg and 8 kg, respectively, collide with initial velocities $(v_A)_1 \sim N(\mu_{A1}, \sigma_{A1}^2) = N(2, 0.1^2)$ m/s and $(v_B)_1 \sim N(\mu_{B1}, \sigma_{B1}^2) = N(3, 0.2^2)$ m/s. If the coefficient of restitution for the disks is $e = 0.8$, determine the x component of the final velocity of each disk just after collision.



Solution:

Resolving the initial velocities into x and y components,

$$(v_{Ax})_1 = (v_A)_1 \cos\theta_1, (v_{Ay})_1 = (v_A)_1 \sin\theta_1$$

$$(v_{Bx})_1 = -(v_B)_1 \cos\phi_1, (v_{By})_1 = -(v_B)_1 \sin\phi_1$$

Conservation of "x" momentum

$$m_A (v_{Ax})_1 + m_B (v_{Bx})_1 = m_A (v_{Ax})_2 + m_B (v_{Bx})_2$$

Coefficient of Restitution (x)

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$$

Thus,

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_A)_1 \cos\theta_1 - \left[-(v_B)_1 \cos\phi_1\right]} = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_A)_1 \cos\theta_1 + (v_B)_1 \cos\phi_1}$$

$$\Rightarrow (v_{Bx})_2 = (v_{Ax})_2 + e\left[(v_A)_1 \cos\theta_1 + (v_B)_1 \cos\phi_1\right]$$

$$m_A (v_A)_1 \cos\theta_1 + m_B \left[-(v_B)_1 \cos\phi_1\right] = m_A (v_{Ax})_2 + m_B (v_{Bx})_2$$
$$= m_A (v_{Ax})_2 + m_B (v_{Ax})_2 + m_B e\left[(v_A)_1 \cos\theta_1 + (v_B)_1 \cos\phi_1\right]$$

$$\Rightarrow (v_{Ax})_2 = \frac{(m_A - m_B e)\cos\theta_1 (v_A)_1 - (1+e)m_B \cos\phi_1 (v_B)_1}{m_A + m_B}$$

Thus, we have

$$\begin{aligned}\mu_{(v_{Ax})_2} &= \frac{(m_A - m_B e)\cos 30^\circ \mu_{A1} - (1+e)m_B \cos 60^\circ \mu_{B1}}{m_A + m_B} \\ &= \frac{(5 - 8\times 0.8)\times \frac{\sqrt{3}}{2}\times 2 - (1+0.8)\times 8\times \frac{1}{2}\times 3}{5+8} \\ &= -1.85 \text{ m/s}\end{aligned}$$

$$\begin{aligned}\mu_{(v_{Bx})_2} &= \mu_{(v_{Ax})_2} + e(\mu_{A1}\cos 30^\circ + \mu_{B1}\cos 60^\circ) \\ &= -1.8481 + 0.8\times(2\times\frac{\sqrt{3}}{2} + 3\times\frac{1}{2}) \\ &= 0.74 \text{ m/s}\end{aligned}$$

$$\begin{aligned}\sigma_{(v_{Ax})_2} &= \sqrt{\left(\frac{(m_A - m_B e)\cos\theta_1}{m_A + m_B}\right)^2 \sigma_{A1}^2 + \left(\frac{-(1+e)m_B\cos\phi_1}{m_A + m_B}\right)^2 \sigma_{B1}^2} \\ &= \sqrt{\left(\frac{(5 - 8\times 0.8)\cos 30^\circ}{5+8}\right)^2 0.1^2 + \left(\frac{-(1+0.8)\times 8\times\cos 60^\circ}{5+8}\right)^2 0.2^2} \\ &= 0.11 \text{ m/s}\end{aligned}$$

$$\begin{aligned}\sigma_{(v_{Bx})_2} &= \sqrt{(\sigma_{(v_{Ax})_2})^2 + (e\cos\theta_1)^2\sigma_{A1}^2 + (e\cos\phi_1)^2\sigma_{B1}^2} \\ &= \sqrt{0.11^2 + \left(0.8\times\frac{\sqrt{3}}{2}\right)^2 0.1^2 + \left(0.8\times\frac{1}{2}\right)^2 0.2^2} \\ &= 0.15 \text{ m/s}\end{aligned}$$

Therefore, $(v_{Ax})_2 \sim N(-1.85, 0.11^2)$ m/s and $(v_{Bx})_2 \sim N(0.74, 0.15^2)$ m/s . **Ans.**