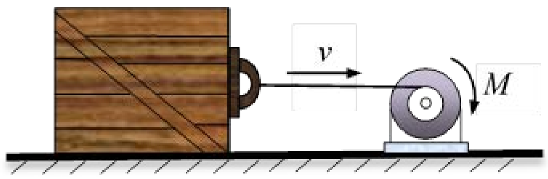
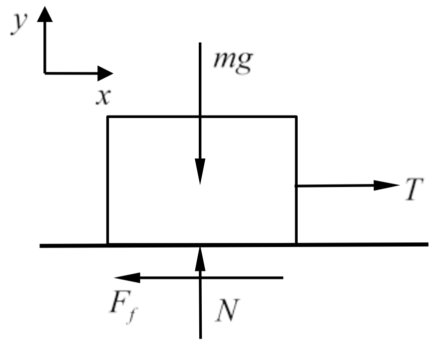
2-2. The motor draws in the cable at a rate of $v = ct^2$ m/s, where c follows a normal distribution $c \sim N(0.2, 0.02^2)$ and t is in seconds. The crate has a mass of 50 kg, and the coefficient of the kinetic friction between the crate and the ground is $\mu_k \sim N(0.3, 0.03^2)$. If the maximum tension that the cable can stand is $T_{\max} = 400\ N$. Assume that c and μ_k are independent. Determine the time when the probability that the cable does not beak is 0.95.



Solution



$$v = ct^2$$
$$a = \frac{dv}{dt} = 2ct$$

Referring to the free-body diagram of the crate, we have

$$\Sigma F_y = ma_y$$
$$N - mg = 0$$
$$N = mg$$

$$\Sigma F_x = ma_x$$
$$T - \mu_k N = m(2ct)$$
$$T = mg\mu_k + 2mct$$

Thus

$$\mu_T = mg\mu_{\mu_k} + 2m\mu_c t$$
$$= 50(9.81)(0.3) + 2(50)(0.2)t$$
$$= 147.15 + 20t$$

$$\sigma_T = \sqrt{(mg\sigma_{\mu_k})^2 + (2m\sigma_c t)^2}$$
$$= \sqrt{\left[50(9.81)(0.03)\right]^2 + \left[2(50)(0.02)t\right]^2}$$
$$= \sqrt{216.53 + 4t^2}$$

$$P(T < T_{\max}) = \Phi\left(\frac{T_{\max} - \mu_T}{\sigma_T}\right) = 0.95$$
$$\frac{T_{\max} - (mg\mu_{\mu_k} + 2m\mu_c t)}{\sqrt{(mg\sigma_{\mu_k})^2 + (2m\sigma_c t)^2}} = \Phi^{-1}(0.95)$$
$$\frac{400 - (147.15 + 20t)}{\sqrt{216.53 + 4t^2}} = \Phi^{-1}(0.95)$$

Solving the above equation

$$t = 10.53\,\text{s}$$ **Ans.**