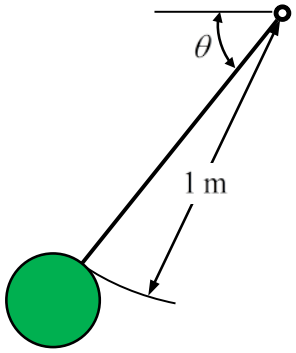
2-20. Due to manufacturing uncertainty, the mass of the ball is $m \sim N(10,1^2)$ kg, and the allowable tension of the rope is $T_A \sim N(140,5^2)$ N. At the instant $\theta = 90^\circ$, the ball has a speed $v = 2$ m/s. Assume m and T_A are independent, determine the probability of failure of the rope. Neglect the mass of the rope and the size of the ball.



$$T - mg\sin\theta = m\frac{v^2}{l}$$

$$T = m\left(g\sin\theta + \frac{v^2}{l}\right)$$

When $\theta = 90^\circ$

$$T = m\left(g + \frac{v^2}{l}\right)$$

Let $Y = T_A - T = T_A - m\left(g + \frac{v^2}{l}\right)$

$$\mu_Y = \mu_{T_A} - \mu_m\left(g + \frac{v^2}{l}\right) = 1.9 \text{ N}$$

$$\sigma_Y = \sqrt{\sigma_{T_A}^2 + \sigma_m^2\left(g + \frac{v^2}{l}\right)^2} = 14.69 \text{ N}$$

$$p_f = \Pr\{Y < 0\} = \Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) = \Phi\left(-\frac{1.9}{14.69}\right) = 0.45$$

The probability of failure is 0.45. **Ans.**