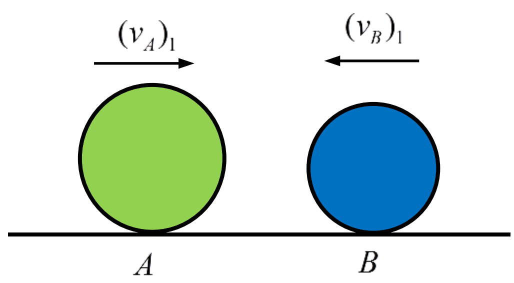
2-28. Before a direct collision, two balls are moving toward each other with normally distributed velocities $(v_A)_1 \sim N(3, 0.3^2)$ m/s and $(v_B)_1 \sim N(5, 0.5^2)$ m/s . The coefficient of restitution between the balls is $e = 0.8$. If the masses of the two balls are $m_A = 4$ kg and $m_B = 3$ kg , respectively, determine the velocity distributions of the two balls just after the collision. Assume $(v_A)_1$ and $(v_B)_1$ are independent.



$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2 \tag{1}$$

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} \tag{2}$$

Solving (1) and (2) yields

$$(v_A)_2 = \frac{(m_A - em_B)(v_A)_1 + m_B(1+e)(v_B)_1}{m_A + m_B}$$

$$(v_B)_2 = \frac{(1+e)m_A(v_A)_1 + (m_B - em_A)(v_B)_1}{m_A + m_B}$$

$$\begin{aligned}\mu_{(v_A)_2} &= \frac{(m_A - em_B)\mu_{(v_A)_1} + m_B(1+e)\mu_{(v_B)_1}}{m_A + m_B} \\ &= \frac{\left(4 - 0.8(3)\right)(3) + 3(1+0.8)(-5)}{4+3} \\ &= -3.17 \text{ m/s}\end{aligned}$$

$$\begin{aligned}\sigma_{(v_A)_2} &= \sqrt{\left(\frac{(m_A - em_B)\sigma_{(v_A)_1}}{m_A + m_B}\right)^2 + \left(\frac{m_B(1+e)\sigma_{(v_B)_1}}{m_A + m_B}\right)^2} \\ &= \sqrt{\left(\frac{\left(4 - 0.8(3)\right)(0.3)}{4+3}\right)^2 + \left(\frac{3(1+0.8)(0.5)}{4+3}\right)^2} \\ &= 0.39 \text{ m/s}\end{aligned}$$

$$\mu_{(v_B)_2} = \frac{(1+e)m_A\mu_{(v_A)_1} + (m_B - em_A)\mu_{(v_B)_1}}{m_A + m_B}$$

$$= \frac{(1+0.8)(4)(3) + \left(3-0.8(4)\right)(-5)}{4+3}$$

$$= 3.23\text{ m/s}$$

$$\sigma_{(v_B)_2} = \sqrt{\left(\frac{(1+e)m_A\sigma_{(v_A)_1}}{m_A+m_B}\right)^2 + \left(\frac{(m_B - em_A)\sigma_{(v_B)_1}}{m_A+m_B}\right)^2}$$

$$= \sqrt{\left(\frac{(1+0.8)(4)(0.3)}{4+3}\right)^2 + \left(\frac{\left(3-0.8(4)\right)(0.5)}{4+3}\right)^2}$$

$$= 0.31\text{ m/s}$$

Therefore, $(v_A)_2 \sim N(3.17, 0.39^2)$ m/s, $\leftarrow$; $(v_B)_2 \sim N(3.23, 0.31^2)$ m/s, $\rightarrow$.