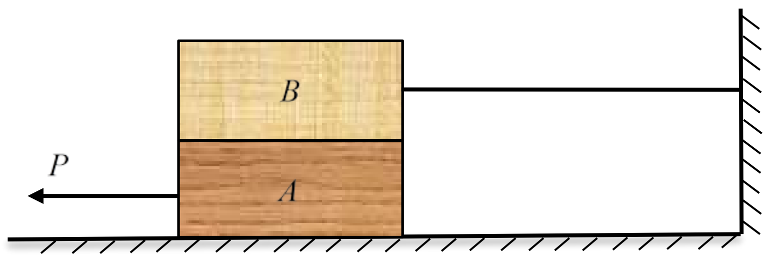
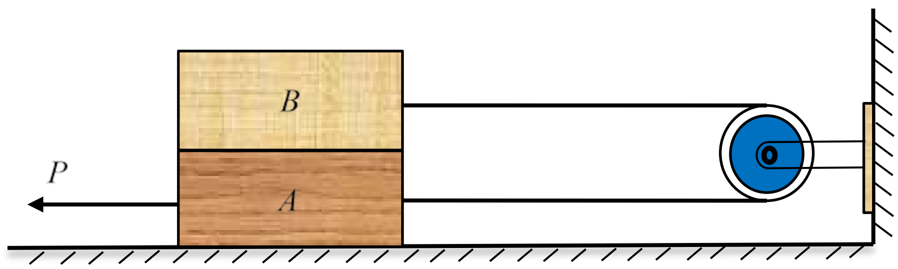
2-3. Each of the two blocks has a mass m. The coefficient of kinetic friction on all surfaces of contact is μ. If a horizontal force P moves the bottom block A, Determine acceleration of block A for the following cases:

(1) m, P, and μ are constants.

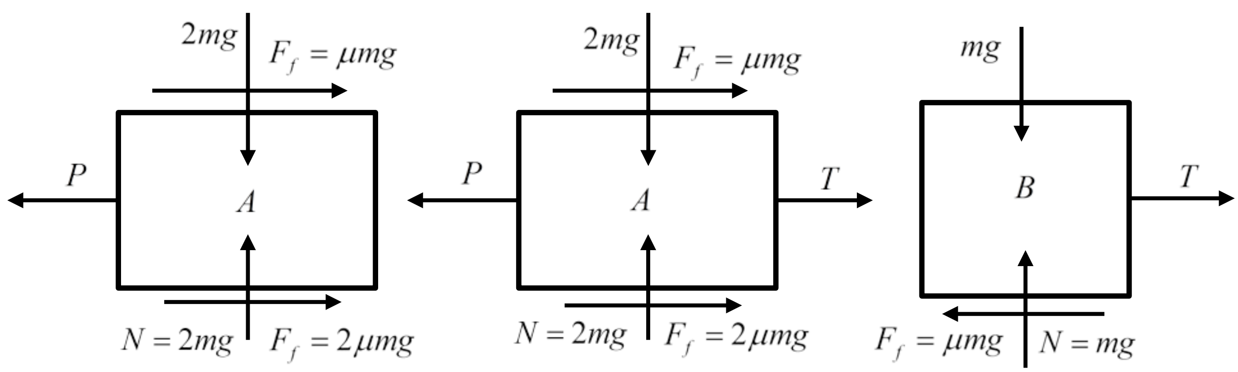
(2) m is a constant while $\mu \sim N(\mu_\mu, \sigma_\mu^2)$ and $P \sim P(\mu_P, \sigma_P^2)$. Assume μ and P are independent.



Case 1



Case 2



Case 1 Case 2 Case 2

(1) When m, P, and μ are constants.

Case 1

Block A:

$$\overset{+}{\leftarrow}\Sigma F_x = ma_x$$
$$P - 3\mu mg = ma_A$$
$$a_A = \frac{P}{m} - 3\mu g$$ **Ans.**

Case 2

Block B:

$$\overset{+}{\leftarrow}\Sigma F_x = ma_x$$
$$\mu mg - T = ma_B$$

Block A:

$$\overset{+}{\leftarrow}\Sigma F_x = ma_x$$
$$P - 3\mu mg - T = ma_A$$

Since $s_A + s_B = l$

$$a_A = -a_B$$

Solving the above equations yields

$$a_A = \frac{P}{2m} - 2\mu g$$ **Ans.**

(2) When $\mu \sim N(\mu_\mu, \sigma_\mu^2)$ and $P \sim P(\mu_P, \sigma_P^2)$

Case 1:

$$a_B = 0$$
$$a_A = \frac{P}{m} - 3\mu g$$
$$\mu_{a_A} = \frac{\mu_P}{m} - 3\mu_\mu g$$ **Ans.**
$$\sigma_{a_A} = \sqrt{\left(\frac{\sigma_P}{m}\right)^2 + (-3\sigma_\mu g)^2}$$ **Ans.**

Thus, a_A is normally distributed with $\mu_{a_A} = \frac{\mu_P}{m} - 3\mu_\mu g$ and $\sigma_{a_A} = \sqrt{\left(\frac{\sigma_P}{m}\right)^2 + (-3\sigma_\mu g)^2}$.

Case 2:

$$a_A = \frac{P}{2m} - 2\mu g$$

$$\mu_{a_A} = \frac{\mu_P}{2m} - 2\mu_\mu g$$ **Ans.**

$$\sigma_{a_A} = \sqrt{\left(\frac{\sigma_P}{2m}\right)^2 + (-2\sigma_\mu g)^2}$$ **Ans.**

Thus, a_A is normally distributed with $\mu_{a_A} = \frac{\mu_P}{2m} - 2\mu_\mu g$ and $\sigma_{a_A} = \sqrt{\left(\frac{\sigma_P}{2m}\right)^2 + (-2\sigma_\mu g)^2}$.