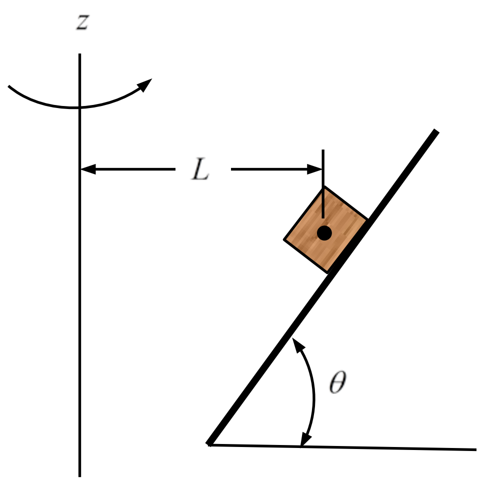
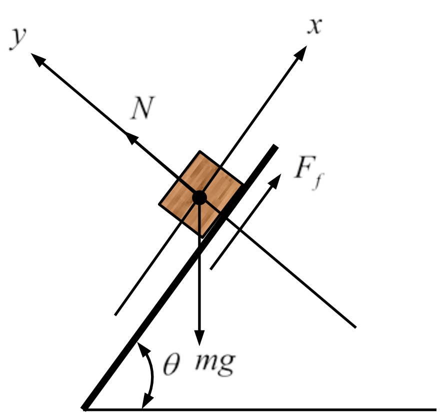
2-33. A 10-kg box lies against the slope and is rotating about the z axis with a constant angular velocity $\omega = 4$ rad/s. The coefficient of static friction between the block and the slope is normally distributed $\mu_s \sim N(0.4, 0.04^2)$. Assume $L = 0.5$ m and $\theta = 60^\circ$, determine the probability that the box will not slip.



Solution: the free body diagram of the block is



$$\Sigma F_y = m(a_n)_y : N - mg\cos\theta = mL\omega^2 \sin\theta$$

$$N = mg\cos\theta + mL\omega^2\sin\theta$$

$$\Sigma F_x = m(a_n)_x : F_f - mg\sin\theta = -mL\omega^2\cos\theta$$

$$F_f = mg\sin\theta - mL\omega^2\cos\theta$$

If $F_f < N\mu_s$, the box will not slip. Let

$$Y = N\mu_s - F_f = \left(mg\cos\theta + mL\omega^2\sin\theta\right)\mu_s - mg\sin\theta + mL\omega^2\cos\theta$$

$$\mu_Y = \left(mg\cos\theta + mL\omega^2\sin\theta\right)\mu_{\mu_s} - mg\sin\theta + mL\omega^2\cos\theta = 2.38\ \text{N}$$

$$\sigma_Y = \left(mg\cos\theta + mL\omega^2\sin\theta\right)\sigma_{\mu_s} = 4.73\ \text{N}$$

$$\Pr\{Y > 0\} = 1 - \Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) = 1 - \Phi\left(-\frac{2.38}{4.73}\right) = 0.69$$