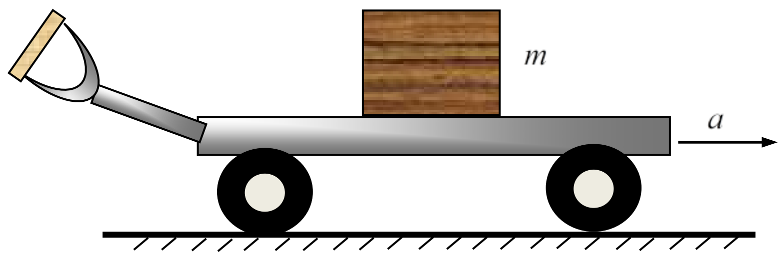
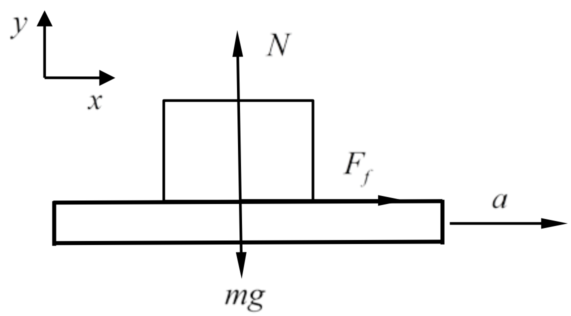
2-4. The mass of the crate is normally distributed with $m \sim N(100, 5^2)$ kg. The cart starts from rest with constant acceleration. When $t = 8$ s, the speed of the cart reaches 50 km/h. Find the probability that the crate will slip. Assume the coefficient of static friction between the cart and the crate is $\mu_s = 0.2$.



Solution



The final velocity of the cart is $v = \left(50 \frac{\text{km}}{\text{h}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 13.89 \text{ m/s}$.

$$v = v_o + at$$

$$a = \frac{v}{t} = \frac{13.89}{8} = 1.74 \text{ m/s}^2$$

Solve for N:

$$\Sigma F_y = ma_y$$
$$N = mg$$

Solve for F_f:

$$\Sigma F_x = ma_x$$
$$F_f = ma$$

The crate will slip if $F_f > F_{\max}$, and $F_{\max} = \mu_s N = \mu_s mg$

Let $$Y = F_{\max} - F_f = \mu_s mg - ma$$

$$\mu_Y = \mu_{F\max} - \mu_{F_f} = \mu_s \mu_m g - \mu_m a = 22.59 \text{ N}$$

$$\sigma_Y = \sqrt{\left(\mu_s \sigma_m g\right)^2 + \left(-\sigma_m a\right)^2} = 13.10 \text{ N}$$

$$\begin{aligned} \mathrm{P}(F_f > F_{\max}) &= \mathrm{P}(Y < 0) \\ &= \Phi\left(\frac{0 - \mu_Y}{\sigma_Y}\right) \\ &= 0.042 \end{aligned}$$

Ans.