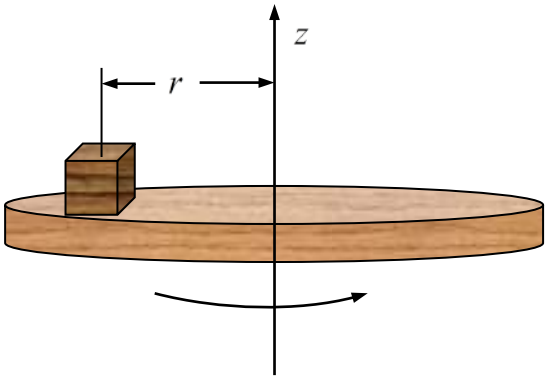
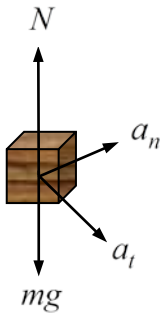
2-5. A block of $m = 10\,\text{kg}$ rests on a horizontal platform at a distance of $r = 3.5$ m from the center of the platform. The angular motion of the platform is slowly increased so that the block's tangential acceleration of the block is negligible. The coefficient of static friction between the block and the platform follows a normal distribution $\mu_s \sim N(0.3, 0.03^2)$. Determine the probability that the block will slip off the platform when its speed is $v = 3\,\text{m/s}$.



Solution



Referring to the free-body diagram of the block, we have

$$\Sigma F_z = 0$$
$$N = mg$$

$$\Sigma F_n = ma_n = m\frac{v^2}{r}$$

When the block is on the verge of slipping,

$$F_f = \mu_s N = \mu_s mg$$

The block will slip off the platform when $F_n > F_f$

Let $$Y = F_f - F_n = \mu_s mg - m\frac{v^2}{r}$$

Therefore,

$$\begin{aligned}\mu_Y &= \mu_{\mu_s} mg - m\frac{v^2}{r} \\ &= 0.3\times 10\times 9.81 - 10\frac{3^2}{3.5} \\ &= 3.72 \text{ N}\end{aligned}$$

$$\begin{aligned}\sigma_Y &= mg\sigma_{\mu_s} \\ &= 10(9.81)(0.03) \\ &= 2.94 \text{ N}\end{aligned}$$

$$\begin{aligned}\Pr(F_n > F_f) &= \Pr(Y < 0) \\ &= \Phi\left(\frac{0-\mu_Y}{\sigma_Y}\right) \\ &= 0.103\end{aligned}$$

Ans.