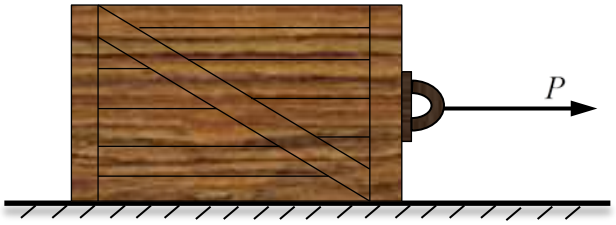
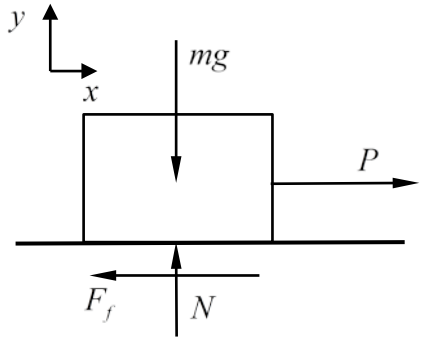
2-6. A crate of $m \sim N(100, 1^2)$ kg starts from rest due to a random force P, after the crate has traveled a distance of $s = 10$ m, its velocity is 5 m/s. If the coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.3$, determine the distribution of P. Assume m and P are independent.



Solution



Free-Body Diagram: Referring to the free-body diagram of the crate, we have

$$F_y = ma_y;\ N - mg = 0 \Rightarrow N = mg$$

$$F_f = u_k N = u_k mg$$

Principle of Work and Energy: Referring to the free-body diagram, only P and F_f do work. P does positive work and F_f does negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + P \cdot s - F_f \cdot s = \frac{1}{2} mv^2$$

$$(P - u_k mg)s = \frac{1}{2} mv^2$$

$$P = \left(\frac{mv^2}{2s} + u_k g \right) m$$

Thus we have

$$\mu_P = \left(\frac{v^2}{2s} + u_k g \right) \mu_m = 419.3 \text{ N}$$

$$\sigma_P = \sqrt{\left(\frac{v^2}{2s} + u_k g \right)^2 \sigma_m{}^2} = 4.19 \text{ N}$$

Therefore, $P \sim N(419.3, 4.19^2)$ N . **Ans.**