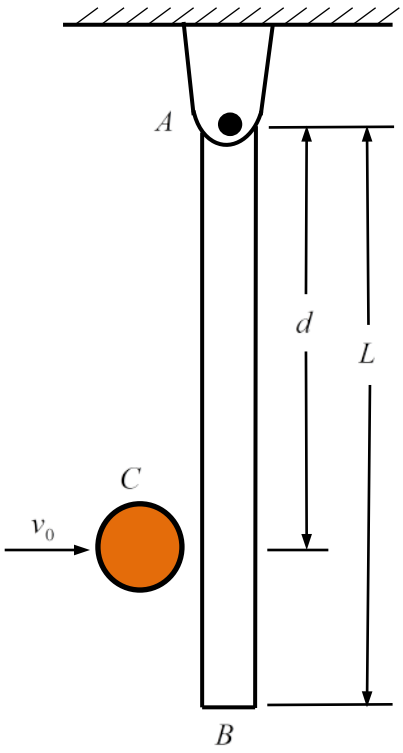
4-1. A rod of weight $w_{AB} = 10$ lb and length $L = 4$ ft is suspended in the vertical position at rest. A ball of weight $w = 2$ lb hits the rod at velocity $v_0 \sim N(30, 2^2)$ ft/s. If $d = 3$ ft, find the distribution of the angular velocity of the rod just after the strike. Take $e = 0.8$.



Solution:

Conservation of impulse

$$(H_A)_1 = (H_A)_2$$

$$mv_0 d = \frac{1}{3} m_{AB} L^2 \omega + mvd$$

At point C

$$v_c = \omega d$$

Thus,

$$e = \frac{v_c - v}{v_0 - 0} = \frac{\omega d - v}{v_0 - 0}$$

$$v = \omega d - v_0 e$$

$$mv_0 d = \frac{1}{3} m_{AB} L^2 \omega + mvd$$

$$= \frac{1}{3} m_{AB} L^2 \omega + m\left(\omega d - v_0 e\right) d$$

$$\omega = \frac{(1+e) m v_0 d}{\frac{1}{3} m_{AB} L^2 + m d^2}$$

thus,

$$\mu_\omega = \frac{(1+e) md}{\frac{1}{3} m_{AB} L^2 + m d^2} \mu_{v_0} = 4.54 \text{ rad/s}$$

$$\sigma_\omega = \frac{(1+e) md}{\frac{1}{3} m_{AB} L^2 + m d^2} \sigma_{v_0} = 0.30 \text{ rad/s}$$

Therefore, $\omega \sim N(4.54, 0.30^2)$ rad/s. **Ans.**