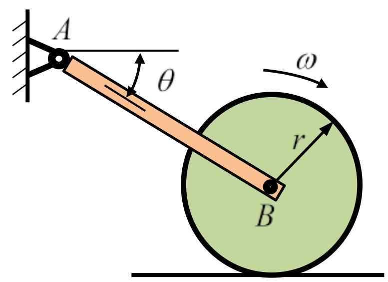
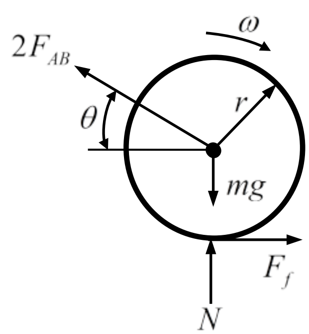
4-12. The 100 kg cylinder has a normally distributed angular velocity $\omega \sim N(50,5^2)$ rad/s when it is brought into contact with the ground. If $r = 0.5$ and $\theta = 45^\circ$, find the probability that the cylinder will stop in 4 seconds. The axle through the cylinder is connected to two symmetrical links. (Only AB is shown). Neglect the weight of the links. The coefficient of kinetic friction is $\mu = 0.4$.





$$m\left(v_{G_y}\right)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m\left(v_{G_y}\right)_2$$

$$(+\uparrow)\ 0 + Nt + 2F_{AB}\sin\theta t - mgt = 0$$

$$m\left(v_{G_x}\right)_1 + \Sigma\int_{t_1}^{t_2} F_x dt = m\left(v_{G_x}\right)_2$$

$$(+\rightarrow)\ 0 + \mu Nt - 2F_{AB}\cos\theta t = 0$$

From above equations, we have

$$N = \frac{mg}{\mu\tan\theta + 1}$$

$$I_G\omega_1 + \Sigma\int_{t_1}^{t_2} M_G dt = I_G\omega_2$$

$$-\frac{1}{2}mr^2\omega_1 + \mu Nrt = 0$$

$$t = \frac{mr\omega_1}{2\mu N} = \frac{(\mu \tan\theta + 1) r\omega_1}{2\mu g}$$

$$\mu_t = \frac{(\mu \tan\theta + 1) r\mu_\omega}{2\mu g} = \frac{(0.4\tan 45^\circ + 1)(0.5)(50)}{2(0.4)(9.81)} = 4.46 \text{ s}$$

$$\sigma_t = \frac{(\mu \tan\theta + 1) r\sigma_\omega}{2\mu g} = \frac{(0.4\tan 45^\circ + 1)(0.5)(5)}{2(0.4)(9.81)} = 0.45 \text{ s}$$

$$\Pr\{t < 4\} = \Phi\left(\frac{4 - \mu_t}{\sigma_t}\right) = \Phi\left(\frac{4 - 4.46}{0.45}\right) = 0.15$$