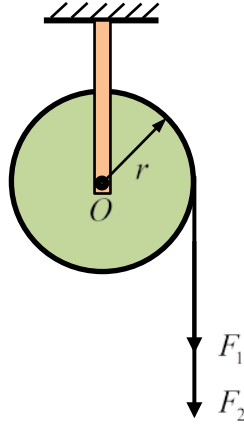


4-15. A 10-kg reel is subjected to two independently and normally distributed force $F_1 \sim N(100, 10^2)$ N and $F_2 \sim N(100, 5^2)$ N. The radius of gyration of the reel about O is $k_G = 8$ m, and $r = 0.6$ m. If the reel starts from rest, determine the probability that the angular velocity of the reel is smaller than 1 rad/s after 5 seconds. Neglect friction and the weight of the rope.



$$(H_o)_1 + \Sigma \int M_o dt = (H_o)_2$$

$$0 + (F_1 + F_2)rt = mk_G^2 \omega$$

$$\omega = \frac{(F_1 + F_2)rt}{mk_G^2}$$

$$\mu_\omega = \frac{(\mu_{F_1} + \mu_{F_2})rt}{mk_G^2} = \frac{(100 + 100)(0.6)(5)}{10(8)^2} = 0.94 \text{ rad/s}$$

$$\sigma_\omega = \frac{\sqrt{\sigma_{F_1}^2 + \sigma_{F_2}^2} rt}{mk_G^2} = \frac{\sqrt{10^2 + 5^2} (0.6)(5)}{10(8)^2} = 0.05 \text{ rad/s}$$

$$\Pr\{\omega < 1\} = \Phi\left(\frac{1 - \mu_\omega}{\sigma_\omega}\right) = \Phi\left(\frac{1 - 0.94}{0.05}\right) = 0.88$$