4-2. A double pulley, with a mass of $m_o \sim N(10,1^2)$ kg and a radius of gyration around its center of $k_o = 0.3$ m, consists of two parts connected to each other. The portion with $r_B = 0.2$ m is connected to block B, $m_B \sim N(10,1^2)$ kg, while the portion with $r_A = 0.4$ m is connected to block A, $m_A \sim N(5,0.5^2)$ kg. If the weight of the cables is negligible and they do not slip, find the kinetic energy of the system when $\omega = 5$ rad/s clockwise.



Solution:

$$T = \frac{1}{2} I_{o} \omega_{o}^{2} + \frac{1}{2} m_{A} v_{A}^{2} + \frac{1}{2} m_{B} v_{B}^{2}$$

$$= \frac{1}{2} (m_{o} k_{o}^{2}) \omega^{2} + \frac{1}{2} m_{A} (\omega r_{A})^{2} + \frac{1}{2} m_{B} (\omega r_{B})^{2}$$

$$= \frac{1}{2} k_{o}^{2} \omega^{2} m_{o} + \frac{1}{2} r_{A}^{2} \omega^{2} m_{A} + \frac{1}{2} r_{B}^{2} \omega^{2} m_{B}$$

$$\mu_{T} = \frac{1}{2} k_{o}^{2} \omega^{2} \mu_{m_{o}} + \frac{1}{2} r_{A}^{2} \omega^{2} \mu_{m_{A}} + \frac{1}{2} r_{B}^{2} \omega^{2} \mu_{m_{B}} = 26.25 \text{ J}$$

$$\sigma_{T} = \sqrt{\left(\frac{k_{o}^{2} \omega^{2}}{2}\right)^{2}} \sigma_{m_{o}}^{2} + \left[\frac{(\omega r_{A})^{2}}{2}\right]^{2} \sigma_{m_{A}}^{2} + \left[\frac{(\omega r_{B})^{2}}{2}\right]^{2} \sigma_{m_{B}}^{2} = 1.59 \text{ J}$$

Therefore, $T \sim N(26.25, 1.59^2) \text{ J}$. Ans.